08

Every object in the universe attracts every other object with a force which is called force of gravitation.

Galileo was the first to recognise the fact that all bodies, irrespective of their masses fall towards the earth with a constant acceleration. Kepler believed all heavenly bodies obey a mathematical order unlike anything on the earth.

GRAVITATION

|TOPIC 1|
Theory of Planetary Motion

In early days, people were observing the heavenly bodies like sun, moon, stars, planets etc., and their movement. It was the Italian physicist **Galileo**, (1564-1642), who recognised all the heavenly bodies like the sun, planets, the moon etc., are moving around the earth which is stationary and was taken as the centre of the universe.

The earliest recorded model for planetary motions proposed by **Ptolemy** about 2000 years ago was a **geocentric model**. It states that the description of the cosmos where the earth is at the orbital centre of all celestial bodies.

However, a more elegant model in which the sun was the centre around which the planet revolved the **heliocentric model**. This theory was later on supported by Galileo from his experimental study on the moon and other planets.

Tycho Brahe (1546-1601) spent his entire lifetime recording observations of the planets with the naked eyes. His assistants Johannes Kepler compiled his data and analysed. He extracted three elegant laws that are now known as Kepler's law.

KEPLER'S LAWS OF PLANETARY MOTION

Kepler's laws of planetary motion are three scientific laws describing motion of planets around the sun.

Kepler's First Law of Orbits

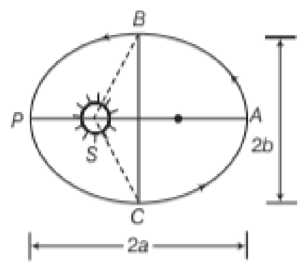
According to this law, all planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.

CHAPTER CHECKLIST

- Theory of Planetary Motion
- Kepler's Laws of Planetary Motion
- Universal Law of Gravitation
- Principle of Superposition
- Gravitational Constant (Cavendish's Experiment)
- Acceleration due to Gravity of Earth
- Gravitational Potential
- Gravitational Potential Energy
- Escape Speed
- Earth Satellites
- Energy of an Orbiting Satellite

This law identifies that the distance between the sun and the earth is constantly changing as the earth goes around its orbit as shown in figure.

It shows an ellipse traced out by a planet around the sun, S. The closest point is P and the farthest point is A. P is called the **perihelion** and A is the **aphelion**. The length of the major axis is equal to the sum of the planet-sun distance at perihelion plus the planet-sun distance at aphelion. The length of the major axis is $2a$. Half the distance AP is length a of the semi-major axis.

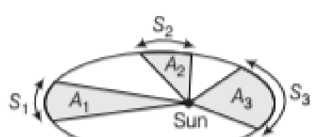


An ellipse traced out by a planet around the sun

Kepler's Second Law of Areas

According to this law, the speed of planet varies in such a way that the radius vector drawn from the sun to a planet sweeps out equal areas in equal interval of time.

i.e. The areal velocity of the planet around the sun is constant.



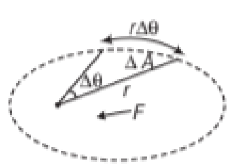
Laws of areas

The elliptical orbit of a planet around the sun is shown in figure. The areas A_1, A_2 and A_3 are swept out by the radius vector in equal intervals of time. According to Kepler's second law i.e. $A_1 = A_2 = A_3$

Note

Areal velocity may be defined as the area swept by the radius vector in unit time.

This law is identical with the law of conservation of angular momentum. Consider a small area ΔA described in a small time interval Δt and let the position of the planet be denoted by r.



Now, $$\Delta A = \frac{1}{2} r (r\,\Delta\theta)$$

$\therefore$ $$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta\theta}{\Delta t}$$

Proceeding to limit as $\Delta t \to 0$, we get

$$\lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

or $$\frac{dA}{dt} = \frac{1}{2} r^2 \omega \qquad \left[\because\ \omega = \frac{\Delta\theta}{\Delta t}\right] \ldots\text{(i)}$$

Instantaneous angular momentum,

$$L = mr^2\omega \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{dA}{dt} = \frac{L}{2m}$$

or $$L = 2m \times \frac{dA}{dt} \qquad \ldots\text{(iii)}$$

The line of action of the gravitational force passes through the axis. Therefore, angle between **r** and **F** is 180°.

Now, $\tau_{\text{ext}} = \mathbf{r} \times \mathbf{F} = rF \sin 180^\circ\,\hat{\mathbf{n}} = 0$

$\therefore$ $\tau_{\text{ext}} = 0$

$$L = \text{constant} \qquad \left[\because\ \frac{dL}{dt} = \tau_{\text{ext}}\right] \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$\therefore$ Areal velocity, $\dfrac{dA}{dt} = \text{constant}$

i.e. **The areal velocity of the planet around the sun is constant.** This proved the Kepler's second law of planetary motion.

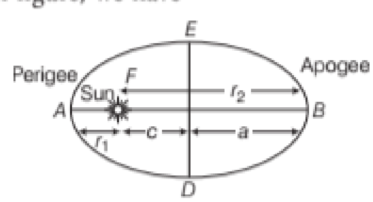
Kepler's Third Law of Period

According to this law, the square of the time period of revolution of a planet around the sun is proportional to the cube of the semi-major axis of its elliptical orbit.

T = time period of revolution of a planet

i.e. $$\boxed{T^2 \propto a^3} \qquad \ldots\text{(i)}$$

As shown in figure, we have



Law of period

$$AB = AF + FB$$

$$2a = r_1 + r_2$$

$\therefore$ $$a = \frac{r_1 + r_2}{2}$$

So, putting the value of a in Eq. (i), we get

$$\boxed{T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3}$$

where, a = length of semi-major axis,

r_1 = shortest distance of planet from sun (perigee)

r_2 = longest distance of planet from sun (apogee).

Note

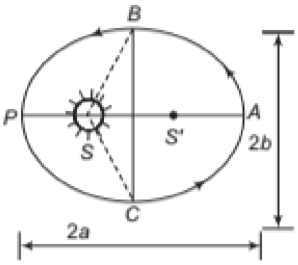
Kepler's laws are applicable not only to the solar system but also to the artificial satellites as well as to the moon going round the planets.

EXAMPLE |1| Angular Momentum of a Planet

Let the speed of the planet at the perihelion P in figure be v_P and the sun-planet distance SP be r_P. Relate $\{r_P, v_P\}$ to the corresponding quantities at the aphelion $\{r_A, v_A\}$. Will the planet take equal time to traverse BAC and CPB? **[NCERT]**

Sol. At any point, radius vector and velocity vector of the planet are mutually perpendicular. Angular momentum of the planet at the perihelion P is

$$L_P = m_P r_P v_P$$



An ellipse traced out by a planet around the sun

Similarly, at the aphelion

$$L_A = m_P r_A v_A$$

By conservation of angular momentum,

$$L_P = L_A$$

or $$m_P r_P v_P = m_P r_A v_A$$

or $$\frac{v_P}{v_A} = \frac{r_A}{r_P} = \frac{a+c}{a-c} = \frac{1+e}{1-e}$$

As, $r_A > r_P$

So, $v_P > v_A$.

Here, $r_P = a - c$

$r_A = a + c$

and eccentricity, $e = \frac{c}{a}$

To traverse BAC, area swept by radius vector = area $SBAC$

To traverse CPB, area swept by radius vector = area $SBPC$

But, area $SBAC$ > area $SBPC$ as shown in figure.

From Kepler's second law, equal areas are swept in equal time. Hence, the planet will take a longer time to traverse BAC than CPB.

Kepler's Law of Period for the Solar System

Planets	Semi-major Axis r (10^{10} m)	Time Period T Years (y)	T^2/r^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165.0	2.99
Pluto	590	248.0	2.99

EXAMPLE |2| Period of Neptune

Calculate the period of revolution of the neptune around the sun, given that diameter of its orbit is 30 times the diameter of the earth's orbit around the sun. Assume both the orbits to be circular.

Sol. According to Kepler's third law,

$$\frac{T_n^2}{T_e^2} = \frac{R_n^3}{R_e^3}$$

where subscripts n and e refer to the neptune and the earth respectively.

$$\therefore \quad T_n^2 = T_e^2 \times \left(\frac{R_n}{R_e}\right)^3 = 1 \times (30)^3$$

[$\because$ time period of the earth's revolution = 1 year and ratio of radii (hence, diameters) of the neptune and the earth is 30]

$$\therefore \quad T_n = 1 \times \sqrt{(30)^3}$$

$$T_n = 30\sqrt{30} = 164.3 \text{ yr}$$

EXAMPLE |3| Orbital Size of a Planet

Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth? **[NCERT]**

Sol. Let T_P and T_E denote the time periods of the planet and the earth, respectively. If R_P and R_E denote the corresponding orbital size as, then

$$\frac{T_P^2}{T_E^2} = \frac{R_P^3}{R_E^3}$$

or $$\left(\frac{R_P}{R_E}\right)^3 = \left(\frac{T_P}{T_E}\right)^2 \Rightarrow \frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{2/3}$$

Since, $$T_P = \frac{1}{2}T_E, \frac{T_P}{T_E} = \frac{1}{2}$$

Thus, $$\frac{R_P}{R_E} = \left(\frac{1}{2}\right)^{2/3} = 0.63$$

Universal Law of Gravitation

It states that

Every body in this universe attracts each other body with a force whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. This force acts along the line joining the centres of two bodies.

Consider two bodies of masses m_1 and m_2 with their centres mutually separated by a distance r as shown in figure. Let F be the force (in magnitude) of gravitational attraction between two bodies.

According to Newton's law of gravitation, we get

Law of gravitation

So, $$F \propto m_1 m_2 \quad \dots(i)$$

and $$F \propto \frac{1}{r^2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

or $$F \propto \frac{m_1 m_2}{r^2}$$

or $$\boxed{\text{Gravitational force, } F = G\frac{m_1 m_2}{r^2}} \quad \dots(iii)$$

where, G is a constant of proportionality known as **gravitational constant**. It is also known as **universal gravitational constant**.

In CGS system, the value of G is 6.67×10^{-8} dyne $\text{cm}^2\,\text{g}^{-2}$ and its SI value is $6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.

Dimensional formula for G

$$G = \frac{F r^2}{m_1 m_2} = \frac{[\text{MLT}^{-2}][\text{L}^2]}{[\text{M}^2]}$$
$$= [\text{MLT}^{-2}][\text{L}^2][\text{M}^{-2}]$$
$$= [\text{M}^{-1}\text{L}^3\text{T}^{-2}]$$

Suppose $m_1 = m_2 = 1$ unit and $r = 1$ unit, then from Eq. (iii), we get

$$F = \frac{G \times 1 \times 1}{(1)^2} \Rightarrow F = G$$

Thus, universal gravitational constant (G) is numerically equal to the force of attraction acting between two bodies, each of unit mass separated by unit distance apart.

Important Points about Gravitational Force

- It is always attractive in nature while electric and magnetic force can be attractive or repulsive.
- It is independent of the intervening medium between the particles while electric and magnetic force depend on the nature of the medium between the particles.
- The gravitational force is a conservative force.
- The value of G does not depend on the nature and size of the masses.
- The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell. Gravitational force possesses spherical symmetry.
- The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside it, is zero.

EXAMPLE |4| Attraction of Sphere

A sphere of mass 40 kg is attracted by a second sphere of mass 60 kg with a force equal to 4 mgf. If G is $6 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$, then calculate the distance between them. Consider acceleration due to gravity is 10m/s^2.

Sol. Given, $M = 40\text{kg}$, $m = 60\text{kg}$,
$$F = 4\,\text{mgf} = 4 \times 10^{-6} \times 10 = 4 \times 10^{-5}\text{N},$$
$$G = 6 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2,\ g = 10\,\text{m/s}^2.$$

According to universal law, $F = \dfrac{GMm}{r^2}$

$$\Rightarrow \quad r = \sqrt{\frac{GMm}{F}} = \sqrt{\frac{6 \times 10^{-11} \times 40 \times 60}{4 \times 10^{-5}}} = 0.06\ \text{m} = 6\ \text{cm}$$

Vector Form of Newton's Law of Gravitation

Consider two particles A and B of masses m_1 and m_2, respectively.

Let $\mathbf{r}_{12}$ = displacement vector from A to B,

$\mathbf{r}_{21}$ = displacement vector from B to A,

$\mathbf{F}_{21}$ = gravitational force exerted on B by A

$\mathbf{F}_{12}$ = gravitational force exerted on A by B

On vector notation, Newton's law of gravitation is written as follows

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21} \quad \dots(i)$$

where, $\hat{\mathbf{r}}_{21}$ is a unit vector pointing towards A. The negative sign indicates that the direction of $\mathbf{F}_{12}$ is opposite to that of $\hat{\mathbf{r}}_{21}$. The negative sign also shows that the gravitational force is attractive in nature.

Similarly, $$\mathbf{F}_{21} = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

where, $\hat{\mathbf{r}}_{12}$ is a unit vector pointing towards B.

But, $$\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$$

Also, $$r_{21}^2 = r_{12}^2$$

$$\therefore \quad \mathbf{F}_{21} = G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21} \quad \ldots\text{(ii)}$$

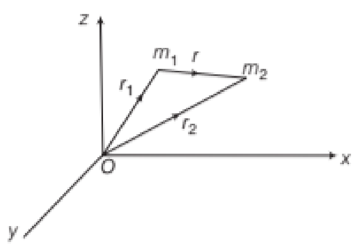
Equating Eqs. (i) and (ii), we have

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

As $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are directed towards the centres of the two particles, so **gravitational force is a central force.**

PRINCIPLE OF SUPERPOSITION

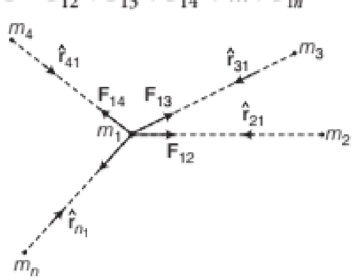
Suppose $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_n$ be the individual forces due to the masses $m_1, m_2, m_3, \ldots, m_n$ which are given by the universal law of gravitation, then from the principle of superposition, each of these forces acts independently and uninfluenced by the other bodies as shown in figure.



Gravitational force on m_1 due to m_2 is along r, where the vector r is $(r_2 - r_1)$

So, the resultant force F can be expressed in vector addition of various forces

$$\mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \ldots + \mathbf{F}_{1n}$$



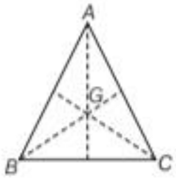
Superposition principle of gravitational forces

Clearly, $$\mathbf{F} = -G\frac{m_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{21} - G\frac{m_1 m_3}{r_{13}^2}\hat{\mathbf{r}}_{31} - \ldots - G\frac{m_1 m_n}{r_{1n}^2}\mathbf{r}_{n1}$$

$$\boxed{\text{Resultant force, } \mathbf{F} = -Gm_1\left[\frac{m_2}{r_{12}^2}\hat{\mathbf{r}}_{21} + \frac{m_3}{r_{13}^2}\hat{\mathbf{r}}_{31} + \ldots \frac{m_n}{r_{1n}^2}\hat{\mathbf{r}}_{n1}\right]}$$

EXAMPLE |5| Force on system of masses

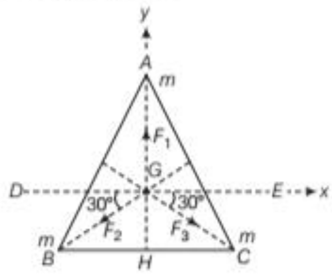
Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC (in figure)



(i) What is the force acting on a mass 2 m placed at the centroid G of the triangle?

(ii) What is the force, if the mass at the vertex A is doubled? (Take, $AG = BG = CG = 1$ m) **[NCERT]**

Sol. In the given figure, $\Delta\, ABC$



Mass of each body A, B and $C = m$

$$F_1 = \frac{G \times m \times 2\,m}{(1)^2} = 2Gm^2 \quad \text{along } GA$$

$$F_2 = \frac{G \times m \times 2m}{(1)^2} = 2Gm^2 \quad \text{along } GB$$

$$F_3 = \frac{G \times m \times 2m}{(1)^2} = 2Gm^2 \quad \text{along } GC$$

Then, $\angle EGC = \angle DGB = 30°$

Resolving F_2 and F_3 into x and y-axes components.

$F_2 \cos 30°$ along GD and $F_2 \sin 30°$ along GH

and $F_3 \cos 30°$ along GE and $F_3 \sin 30°$ along GH

Resultant force on the mass $2m$ at G.

Force at G is $F_1 - (F_2 \sin 30° + F_3 \sin 30°)$

$$F = 2Gm^2 - \left(2Gm^2 \times \frac{1}{2} + 2Gm^2 \times \frac{1}{2}\right) = 0$$

Gravitational force on mass 2 m at G due to mass $2m$ at A.

Net force, $F_1{}'$ is $G\dfrac{2m \times 2m}{1^2} = 4Gm^2$ along GA

Net force acting at G due to masses A, B and C

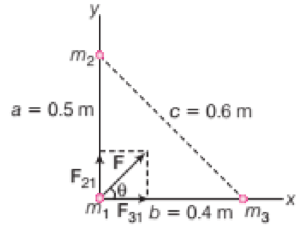
$$= F_1{}' - (F_2 \sin 30° + F_3 \sin 30°)$$

$$= 4\,Gm^2 - \left(2Gm^2 \times \frac{1}{2} + 2Gm^2 \times \frac{1}{2}\right)$$

$$= 2\,Gm^2 \text{ along } GA$$

EXAMPLE |6| Three Balls Positioned at the Corners of a Right Triangle

Consider the three billiard balls of masses 400 g are placed on a table at the corners of a right angle as shown in figure. Calculate the gravitational force vector on the ball of the mass m_1 resulting from other two balls. Also, find the magnitude and direction of this force.



The cue ball is attracted to other balls by the gravitational force. We can see graphically, the net force should point upward and towards right. Hence, we locate our coordinate axes by placing our origin at the position of the one ball as shown in figure.

Sol. Force exerted by m_2 on the cue ball of mass m_1.

$$m_1 = m_2 = m_3 = 400\text{ g} = 0.4\text{ kg}$$

$$a = 0.5\text{ m},\ b = 0.4\text{ m and } c = 0.6\text{ m}$$

Gravitational constant,

$$G = 6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$$

The force

$$\mathbf{F}_{21} = G\frac{m_2 m_1}{a^2}\hat{\mathbf{j}} = \frac{6.67\times10^{-11}\times0.4\times0.4}{(0.5)^2}\cdot\hat{\mathbf{j}}$$

$$\mathbf{F}_{21} = 4.268\times10^{-11}\ \hat{\mathbf{j}}\text{ N}$$

Similarly, $$\mathbf{F}_{31} = \frac{Gm_3 m_1}{b^2}\hat{\mathbf{i}} = \frac{6.67\times10^{-11}\times0.4\times0.4}{(0.4)^2}\hat{\mathbf{i}}$$

$$\mathbf{F}_{31} = 6.67\times10^{-11}\ \hat{\mathbf{i}}\text{ N}$$

The net gravitational force on the one ball,

$$\mathbf{F} = \mathbf{F}_{31} + \mathbf{F}_{21} = (6.67\ \hat{\mathbf{i}} + 4.26\ \hat{\mathbf{j}})\times10^{-11}\text{ N}$$

The net gravitational force,

$$F = \sqrt{\mathbf{F}_{31}^2 + \mathbf{F}_{21}^2} = \sqrt{(6.67)^2 + (4.268)^2}\times10^{-11}\text{ N}$$

$$F = 7.918\times10^{-11}\text{ N}$$

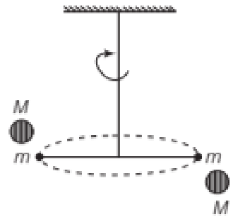
Also, $$\tan\theta = \frac{F_y}{F_x} = \frac{\mathbf{F}_{21}}{\mathbf{F}_{31}} = \frac{4.268\times10^{-11}}{6.67\times10^{-11}} = 0.639$$

$$\Rightarrow\quad \theta = 32.6^\circ$$

GRAVITATIONAL CONSTANT (CAVENDISH'S EXPERIMENT)

The value of the gravitational constant G was first determined experimentally by English Scientist Henry Cavendish in 1798.

Two small lead spheres of mass m each are connected to the end of a rod of length L which is suspended from its mid-point by a fine quartz fibre, forming a torsion balance.



Two large lead spheres of mass M each are brought close to small ones but on opposite sides.

As, the small spheres move towards the larger ones under the gravitational attraction. Thus, the gravitational force on each small sphere due to big sphere is $F = \dfrac{GMm}{R^2}$,

where R is the distance between the centre of the large and its neighbouring small sphere.

Due to this force, the resultant gravitational force on the bar is zero but a gravitational torque acts on it. The value of gravitational torque on the bar

$$= \text{gravitational force}\times\text{length of the bar} = \frac{GMm}{R^2}\times L$$

Due to this gravitational torque, the bar is rotated through an angle θ, say about the wire as an axis and the wire gets twisted through an angle θ. If τ is the restoring torque per unit twist of the thin wire used, then restoring torque $= \tau\,\theta$. In equilibrium position,

Restoring torque = gravitational torque, $\tau\,\theta = \dfrac{GMm}{R^2}\times L$

$$\boxed{\text{Gravitational constant, } G = \frac{R^2\tau\theta}{MmL}}$$

Knowing all the quantities on the right hand side from the experiment, the value of G can be determined. Since, Cavendish's experiment, the measurement of G has been improved upon. The currently accepted value of G is

$$G = 6.67\times10^{-11}\ \text{N - m}^2/\text{kg}^2$$

Newton's Principia

Kepler had formulated his third law by 1619. The announcement of the underlying universal law of gravitation came about seventy years later with the publication in 1687 of Newton's masterpiece Philosophiae Naturalis Principia Mathematica, often simply called the Principia.

Note

G is an universal constant and its value does not vary with change in intervening medium or temperature or pressure, etc.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. For a satellite in elliptical orbit which of the following quantities does not remain constant?

(a) Angular momentum
(b) Linear momentum
(c) Areal velocity
(d) All of the above

Sol. (b) In elliptical orbit, velocity keeps on changing both in magnitude and direction. Therefore, momentum does not remain constant $(P = mv)$.

2. Both the earth and the moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon **[NCERT Exemplar]**

(a) will be elliptical
(b) will not be strictly elliptical because the total gravitational force on it is not central
(c) is not elliptical but will necessarily be a closed curve
(d) deviates considerably from being elliptical due to influence of planets other than the earth

Sol. (b) As observed from the sun, two types of forces are acting on the moon one is due to gravitational attraction between the sun and the moon and the other is due to gravitational attraction between the earth and the moon. Hence, total force on the moon is not central.

3. The velocity of the planet when it is closest to sun is

(a) maximum
(b) minimum
(c) can have any value
(d) None of the above

Sol. (a) From conservation of angular momentum,

$$\text{Velocity of planet}\,(v) \propto \frac{1}{\text{Distance of the planet from sun}\,(r)}$$

So, r_P is minimum for perihelion (P).

$\Rightarrow v_P$ is maximum.

4. Two sphere of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(a) F (b) $\frac{F}{3}$

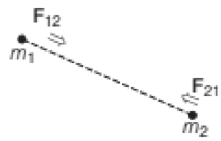
(c) $\frac{F}{9}$ (d) $3F$

Sol. (a) Gravitational force does not depend on the medium between the masses. So, it will remain same i.e., F.

5. If the gravitation force on body 1 due to 2 is given by $\mathbf{F}_{12}$ and on body 2 due to 1 is given as $\mathbf{F}_{21}$, then

(a) $\mathbf{F}_{12} = \mathbf{F}_{21}$
(b) $\mathbf{F}_{12} = -\mathbf{F}_{21}$
(c) $\mathbf{F}_{12} = \frac{\mathbf{F}_{21}}{4}$
(d) None of the above

Sol. (b) Since, gravitational forces are attractive $\mathbf{F}_{12}$ is directed opposite to $\mathbf{F}_{21}$ and they are also equal in magnitude.



Hence, $\mathbf{F}_{21} = -\mathbf{F}_{12}$

or $\mathbf{F}_{12} = -\mathbf{F}_{21}$

VERY SHORT ANSWER Type Questions

6. How earth retains most of the atmosphere?

Sol. Earth retains most of the atmosphere due to force of gravity.

7. If earth be at one half its present distance from the sun, then how many days will there be in a year?

Sol. According to Kepler's third law

$$T^2 \propto R^3$$

Hence, $$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$\Rightarrow \quad T_2^2 = \left[\frac{R_2}{R_1}\right]^3 T_1^2$$

$$\Rightarrow \quad T_2 = T_1\left[\frac{R_2}{R_1}\right]^{3/2} = 365\left(\frac{R/2}{R}\right)^{3/2}$$

$$= 365 \times \frac{1}{2\sqrt{2}} = 129 \text{ days}$$

8. By which law is the Kepler's law of areas identical?

Sol. The law of conservation of angular momentum.

9. Do the force of friction and other contact forces arise due to gravitational attraction? If not, then what is the origin of these forces?

Sol. Contact forces have electrical region.

10. Is the Kepler's law kinematic?

Sol. Yes, because kepler's third law is the relation between distance and time.

11. Why does tides arise in the oceans?

Sol. Tides arise in the oceans due to the force of attraction between the moon and sea water.

12. Assume that the law of gravitation changes from inverse square to inverse cube. Does the angular momentum of a planet about the sun will remain constant?

Sol. The torque on a planet is zero so long as the force is a central force. The angular momentum of planet remains constant so long as torque remains zero.

13. You can shield a change from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means? **[NCERT]**

Sol. No, while electrical forces depend upon the medium, the gravitational forces do not depend upon medium. To sum up, the 'gravity screens' are not possible.

14. If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that sun's pull is greater than the moon's pull. However, the tidal effect of the moon's pull is greater than the tidal effect of the sun. Why? **[NCERT]**

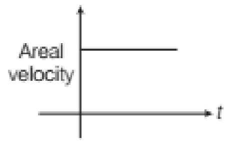
Sol. The tidal effect depends inversely on the cube of the distance unlike force which depends inversely on the square of the distance. As the moon is closer to earth than the sun, so its tidal effect is greater than that of the sun.

The ratio of these two effects is

$$\frac{T_m}{T_s} = \left(\frac{d_s}{d_m}\right)^3 = \left(\frac{1.5\times10^{11}}{3.8\times10^{8}}\right)^3 = 61.5\times10^{6}$$

15. Draw areal velocity versus time graph for mars. **[NCERT Exemplar]**

Sol. Areal velocity of planet revolving around the sun is constant with time (kepler's second law).



16. What is the direction of areal velocity of the earth around the sun? **[NCERT Exemplar]**

Sol. It is normal to the plane containing the earth and the sun as areal velocity.

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r \times v\Delta t$$

and directed according to right hand rule.

17. Out of aphelion and perihelion, where is the speed of the earth more and why? **[NCERT Exemplar]**

Sol. At perihelion, because the earth has to cover greater linear distance to keep the areal velocity constant.

18. From Kepler's second law and observations of the sun's motion as seen from the earth, we can conclude that the earth is closer to the sun during winter in the Northern hemisphere than during summer. Explain.

Sol. The earth is closer to the sun during winter. But heating effect is less because the sun's rays fall obliquely.

19. At what factor between the two particles gravitational force does not depend?

Sol. Gravitational force does not depend upon the medium between the two particles.

20. Are the Kepler's laws applicable only to the solar system?

Sol. No, they are applicable to the artificial satellites too.

21. Two particles of masses m_1 and m_2 attract each other gravitationally and are set in motion under the influence of the gravitational force? Will the centre of mass move?

Sol. Since, gravitational force is an internal force, therefore the centre of mass would not move.

22. When will the Kepler's law be applicable on the planets?

Sol. Kepler's law will be applicable whenever inverse square law is involved.

23. Work done in moving a particle round a closed path under the action of gravitation force is zero. Why?

Sol. Gravitational force is a conservative force which means that work done by it, is independent of path followed.

SHORT ANSWER Type Questions

24. Imagine what would happen if the value of G becomes

(i) 100 times of its present value.

(ii) $\frac{1}{100}$ times of its present value.

Sol. (i) Earth's attraction would be so large that you would be crushed to the earth.

(ii) Earth's attraction would be so less that we can easily jump from the top of a multi-storey building.

25. In Kepler's law of periods, $T^2 = kr^3$, the constant, $k = 10^{-13}\ s^2m^{-3}$. Express the constant k in days and kilometres. The moon is at a distance of 3.84×10^5 km from the earth. Obtain its time period of revolution in days. [NCERT]

Sol. Given, $k = 10^{-13} s^2/m^3$

As, $1\ s = \frac{1}{24 \times 60 \times 60}$ day and $1m = \frac{1}{1000}$ km

$\therefore \quad k = 10^{-13} \times \frac{1}{(24 \times 60 \times 60)^2}(\text{day})^2 \frac{1}{(1/1000)^3}\text{km}^{-3}$

$= 1.33 \times 10^{-14}\ (\text{day})^2\ \text{km}^{-3}$

For the moon, $r = 3.84 \times 10^5$ km

$\therefore \quad T^2 = kr^3 = 1.33 \times 10^{-14} \times (3.84 \times 10^5)^3 = 753.087$

$T = 27.3$ days

26. The time period of a satellite of earth is 5 h. If the separation between the earth and the satellite is increased to four times the previous value, then what will be the new time period?

Sol. $T_2 = T_1\left[\frac{R_2}{R_1}\right]^{3/2} = T_1[4]^{3/2} = 8T_1 = 40\text{h}$

27. A planet moving along an elliptical orbit is closest to the Sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then find the ratio $\frac{v_1}{v_2}$.

Sol. From the law of conservation of angular momentum

$$mr_1v_1 = mr_2v_2$$

$$\Rightarrow \quad r_1v_1 = r_2v_2 \quad \text{or} \quad \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

28. The gravitational force between two spheres is x when the distance between their centres is y. What will be the new force, if the separation is made $3y$?

Sol. $F \propto \frac{1}{r^2}$. So, if r is increased by a factor of 3, F will be reduced by a factor of 9. Thus, the new force will be $\frac{x}{9}$.

29. A mass M is broken into two parts, m and $(M - m)$. How is m related to M so that the gravitational force between two parts is maximum?

Sol. Let $m_1 = m$, $m_2 = M - m$

$$F = G\frac{m(M-m)}{r^2} = \frac{G}{r}(Mm - m^2)$$

Differentiating w.r.t m, $\frac{dF}{dm} = \frac{G}{r^2}(M - 2m)$

for F to be maximum, $\frac{dF}{dm} = 0$

$$\frac{G}{r^2}(M - 2m) = 0$$

$$M = 2m, \text{ or } m = \frac{M}{2}$$

$$\therefore \quad m_1 = m_2 = M/2$$

30. A mass of 1 g is separated from another mass of 1 g by a distance of 1 cm. How many g-wt of force exists between them?

Sol. $F = G\frac{m_1m_2}{r^2}$

$= (6.67 \times 10^{-8})\left(\frac{1 \times 1}{1^2}\right)$ dyne

$= 6.67 \times 10^{-8}$ dyne $= \frac{6.67 \times 10^{-8}}{980}$

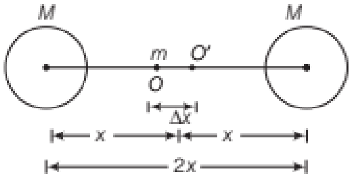
$= 7 \times 10^{-11}$ g-wt

LONG ANSWER Type I Questions

31. Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the mid-point of the line joining their centres be in stable equilibrium or unstable equilibrium? Give reasons for your answer. [NCERT Exemplar]

Sol. If F_L and F_R are the forces exerted on mass m lying at O (after displacing it from O to O'), by masses M lying on left and right, respectively. Then, from figure,

$$F_L = G\frac{Mm}{(x + \Delta x)^2} \quad \text{and} \quad F_R = G\frac{Mm}{(x - \Delta x)^2}$$



Since $F_R > F_L$, the net force on m is towards right. Hence, the equilibrium is unstable.

32. A geostationary satellite is orbiting the earth at a height of $5R$ above the surface of the earth, R being the radius of the earth. Find the time period of another satellite (in hours) at a height of $2R$ from the surface of the earth.

Sol. From Kepler's third law, $T^2 \propto r^3$

Hence, $\quad T_1^2 \propto r_1^3$ and $T_2^2 \propto r_2^3$

So, $$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \frac{(3R)^3}{(6R)^3}$$

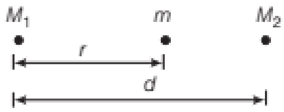
$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{1}{2\sqrt{2}} \qquad [\because T_1 = 12]$$

$$\therefore \quad T_2 = \frac{12}{2\sqrt{2}} = \frac{6}{\sqrt{2}}$$

33. Two stationary particles of masses M_1 and M_2 are a distance d apart. A third particle lying on the line joining the particles, experiences no resultant gravitational force. What is the distance of this particle from M_1?

Sol. The force on m towards M_1 is $F = G\dfrac{M_1 m}{r^2}$

The force on m towards M_2 is $F = G\dfrac{M_2 m}{(d-r)^2}$



Equating two forces, we have

$$G\frac{M_1 m}{r^2} = G\frac{M_2 m}{(d-r)^2}$$

$$\left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1} \quad \text{or} \quad \frac{d}{r} - 1 = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

$$\Rightarrow \qquad \frac{d}{r} = \frac{\sqrt{M_2} + \sqrt{M_1}}{\sqrt{M_1}}$$

So, distance of an particle from m is

$$r = d\left(\frac{\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}\right)$$

34. A saturn year is 29.5 times the earth year. How far is the saturn from the sun of the earth is 1.50×10^8 km away from the sun? **[NCERT]**

Sol. According to Kepler's third law of planetary motion,
$T^2 \propto r^3$

Thus, $$\frac{T_S^2}{T_E^2} = \frac{r_S^3}{r_E^3} \quad \text{or} \quad \left(\frac{T_S}{T_E}\right)^2 = \left(\frac{r_S}{r_E}\right)^3$$

or $$\left(\frac{r_S}{r_E}\right) = \left(\frac{T_S}{T_E}\right)^{2/3}$$

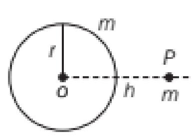
or $$r_S = \left(\frac{T_S}{T_E}\right)^{2/3} \times r_E$$

As, $\dfrac{T_S}{T_E} = 29.5$ and $r_E = 1.5 \times 10^8$ km,

$$r_S = (29.5)^{2/3}(1.5 \times 10^8 \text{ km})$$
$$= 14.3 \times 10^8 \text{ km}$$

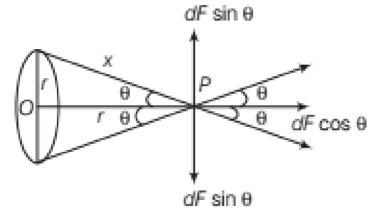
LONG ANSWER Type II Questions

35. A mass m is placed at P, a distance h along the normal through the centre O of a thin circular ring of mass M and radius r. If the mass is removed further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h = r$? **[NCERT Exemplar]**



Sol. Consider a small element of the ring of mass dM, gravitational force between dM and m, distance x apart in figure i.e.

$$dF = \frac{G(dM)m}{x^2}$$



dF can be resolved into two rectangular components.

(i) $dF \cos\theta$ along PO and

(ii) $dF \sin\theta$ perpendicular to PO (given figure)

The total force (F) between the ring and mass (m) can be obtained by integrating the effects of all the elements forming the ring, whereas all the components perpendicular to PO cancel out, i.e. $\int dF \sin\theta = 0$, the component along PO add together to give F i.e.

$$F = \int dF \sin\theta = \int \frac{G(dM)m}{x^2}\left(\frac{h}{x}\right) = \frac{Gmh}{x^3}\int dM$$

i.e. $$F = \frac{GMmh}{(r^2 + h^2)^{3/2}}$$

$$[\because \int dM = M \text{ and } x = (r^2 + h^2)^{1/2}]$$

Since, $h = r$, $F = GMm\left(\dfrac{r}{2\sqrt{2}r^3}\right) = \dfrac{GMm}{2\sqrt{2}r^2}$

When h becomes $2r$, $F' = \dfrac{GMm(2r)}{(r^2 + 4r^2)^{3/2}} = \dfrac{2GMm}{5\sqrt{5}r^2}$

Thus, $\dfrac{F'}{F} = \left(\dfrac{2GMm}{5\sqrt{5}r^2}\right)\left(\dfrac{2\sqrt{2}r^2}{GMm}\right) = \dfrac{4\sqrt{2}}{5\sqrt{5}}$

36. Earth's orbit in an ellipse with eccentricity 0.016. Thus, earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of solar day is not constant through the year. Assume that the earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain variation of length of the day during the year? **[NCERT Exemplar]**

Sol. From the geometry of the ellipse of eccentricity e and semi-major axis a, the aphelion and perihelion distances are

$$r_a = a(1+e)$$
$$r_p = a(1-e)$$

or $$\frac{r_a}{r_p} = \frac{1+e}{1-e}$$

Since, angular momentum $(mr^2\omega)$ is conserved.

$$r^2\omega = \text{constant i.e. } r_p^2\omega_p = r_a^2\omega_a$$

or $$\frac{\omega_p}{\omega_a} = \frac{r_a^2}{r_p^2} = \left(\frac{1+e}{1-e}\right)^2$$

$$= \left(\frac{1+0.0167}{1-0.0167}\right)^2 = 1.0691$$

or $$\left(\frac{\omega_p}{\omega}\right)\left(\frac{\omega}{\omega_a}\right) = 1.0691 \qquad \text{...(i)}$$

where, ω is the angular speed corresponding to mean solar day and can be considered to be the geometric mean of ω_p and ω_a.

i.e. $$\omega_p \cdot \omega_a = \omega^2 \quad \text{or} \quad \frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} = \sqrt{1.0691} = 1.034$$

If $\omega = 1°/$ day (mean angular speed)

$$\omega_p = 1.034°/\text{day and } \omega_a = \frac{1°}{1.034}/\text{day} = 0.967°/\text{day}$$

Since 361° correspond to 24 h, $(360 + 1.034)°$ corresponds to 24.0023 h, i.e. 24 h 8.14″ and $(360 + 0.967)°$ corresponds to 23 h 59′ m 52″.

This does not explain the actual variation in the length of the day during the year.

37. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun $= 2 \times 10^{30}$ kg, mass of the earth $= 6 \times 10^{24}$ kg. Neglect the effect of other planets etc. (Orbital radius $= 1.5 \times 10^{11}$ m). **[NCERT]**

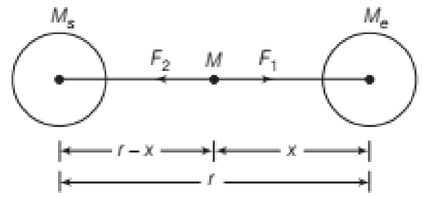
Sol. We are given that

Mass of the sun, $M_s = 2 \times 10^{30}$ kg

Mass of the earth, $M_e = 6 \times 10^{24}$ kg

Distance between the centres of the sun and the earth,

$$r = 1.5 \times 10^{11} \text{ m}$$



Let x be the distance of the required point from the centre of the earth. Clearly, at this point the gravitational force (F_1) on the rocket of mass m due to the earth = gravitational force (F_2) on the rocket due to the sun, i.e. $F_1 = F_2$.

or $$\frac{GmM_e}{x^2} = \frac{GmM_s}{(r-x)^2}$$

or $$\frac{(r-x)^2}{x^2} = \frac{M_s}{M_e} = \frac{2 \times 10^{30} \text{ kg}}{6 \times 10^{24} \text{ kg}} = \frac{10^6}{3}$$

or $$\frac{r-x}{x} = \frac{10^3}{\sqrt{3}} \quad \text{or} \quad \frac{r}{x} - 1 = \frac{10^3}{\sqrt{3}}$$

or $$\frac{r}{x} = \frac{10^3}{\sqrt{3}} + 1 \approx \frac{10^3}{\sqrt{3}}$$

or $$x = \frac{\sqrt{3}r}{10^3} = \frac{1.732(1.5 \times 10^{11}) \text{ m}}{10^3}$$

$$= 2.6 \times 10^8 \text{ m}$$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. According to Kepler's law of planetary motion, if T represents time-period and r is orbital radius, then for two planets these are related as

(a) $\left(\frac{T_1}{T_2}\right)^3 = \left(\frac{r_1}{r_2}\right)^2$
(b) $\left(\frac{T_1}{T_2}\right)^{3/2} = \frac{r_1}{r_2}$
(c) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$
(d) $\left(\frac{T_1}{T_2}\right) = \left(\frac{r_1}{r_2}\right)^{2/3}$

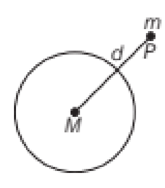
2. As observed from the earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from the earth, this would **[NCERT Exemplar]**

(a) be similarly true
(b) not be true because the force between the earth and mercury is not inverse square law
(c) not be true because the major gravitational force on mercury is due to the sun
(d) not be true because mercury is influenced by forces other than gravitational forces

3. Law of areas is valid only when gravitational force is

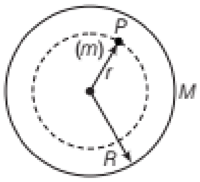
(a) conservative force
(b) central force
(c) attractive force
(d) weak force

4. A point mass m is placed outside a hollow spherical shell of mass M and uniform density at a distance d from centre of the big sphere. Gravitational force on point mass m at P is



(a) $\frac{GmM}{d^2}$ (b) Zero
(c) $\frac{2\,GmM}{d^2}$ (d) Data insufficient

5. The force of attraction due to a hollow spherical shell of mass M, radius R and uniform density, on a point mass m situated inside it is



(a) $\frac{GmM}{r^2}$ (b) $\frac{Gm\,M}{R^2}$
(c) Zero (d) Data insufficient

Answer

1. (c)	2. (c)	3. (b)	4. (a)	5. (c)

VERY SHORT ANSWER Type Questions

6. What would happen to an orbiting planet if all of a sudden it comes to stand still?

7. We cannot move fingers without disturbing all stars, why?

8. In case of earth, at what points is its gravitational field zero?

9. How could we determine the mass of a planet such as venus which has no moon?

SHORT ANSWER Type Questions

10. If the earth be at one half of its present distance from the sun, then how many days will be there in a year? [**Ans.** 129 days]

11. Calculate the force of attraction between two bodies, each of mass 100 kg and 1 m apart on the surface of the earth. [**Ans.** 6.67×10^{-7} N]

LONG ANSWER Type I Questions

12. The distances of two planets from the sun are 10^{13} m and 10^{12} m, respectively. Calculate the ratio of time period and the speeds of the two planets. $\left[\textbf{Ans. } 10\sqrt{10}, \frac{1}{\sqrt{10}}\right]$

13. If the earth is $\frac{1}{4}$ of its present distance from the sun, then what is the duration of the year? [**Ans.** 0.125 yr]

14. A sphere of mass 40 kg is attracted by another sphere of mass 15 kg, with a force of $\frac{1}{40}$ mg-wt. Find the value of the gravitational constant if their centres are 0.40 m apart. [**Ans.** 6.54×10^{-11} Nm^2 kg^{-2}]

15. The masses and coordinates of the three spheres are as follows; 20 kg, $x = 0.50$ m, $y = 1.0$ m, 40 kg, $x = -1.0$ m, $y = -1.0$ m, 60 kg, $x = 0$ m, $y = -0.50$ m. What is the magnitude of the gravitational force on a 20 kg sphere located at the origin due to the other spheres? [**Ans.** 3.2×10^{-7} N]

LONG ANSWER Type II Question

16. Consider two solid uniform spherical objects of the same density ρ. One has a radius R and the other a radius $2R$. They are in outer space where the gravitational fields from other objects are negligible. If they are at rest with their surfaces touching, then what is the contact force between the objects due to their gravitational attraction? $\left[\textbf{Ans. } \frac{128}{81} G\pi^2 R^4 \rho^2\right]$

|TOPIC 2|

Acceleration Due to Gravity and Gravitational Potential Energy

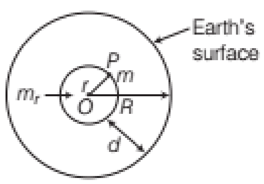
ACCELERATION DUE TO GRAVITY OF EARTH

The force of gravity acting on a body having unit mass placed on or near the surface of the earth is known as **acceleration due to gravity**. In other words, acceleration set up in the body when it falls freely under the effect of gravity is also known as acceleration due to gravity. It is represented by symbol g and its value is 9.8 m/s^2 on the surface of the earth.

At a given place, the value of acceleration due to gravity is the same for all bodies irrespective of their masses. However, it differs from place to place on the surface of the earth. It also varies with altitude and depth.

Hence, at a point outside the earth,the gravitational force is just as if the entire mass of the earth is concentrated at its centre.

e.g. Consider the earth to be made up of concentric shells and a point mass m situated at a distance r from the centre. The point P lies outside the sphere of radius r and the point P lies inside if the shell's radius is greater than r. The smaller sphere exerts a force on a mass m at P as if its mass m_r is concentrated at the centre. The force on the mass m at P has a magnitude



Determination of acceleration due to gravity of earth

$$F = \frac{Gm(m_r)}{r^2} \quad \ldots\text{(i)}$$

Mass on the earth is $M = \frac{4\pi}{3} R^3 \rho$

where, M = mass of the earth, R = radius of the earth and ρ = density of the earth.

Mass on sphere is $m_r = \frac{4\pi}{3} \rho r^3$

Substituting the value of m_r in Eq. (i), we have

$$F = \frac{Gm}{r^2} \frac{4\pi}{3} \rho r^3 = Gm \frac{M}{R^3} r$$

$$\Rightarrow \quad F = G\frac{Mm}{R^2} \quad [\because r = R]$$

From Newton's second law, we know, $F = mg$

$$\therefore \quad g = \frac{F}{m} = \frac{GM}{R^2}$$

$$\boxed{\text{Acceleration due to gravity, } g = \frac{GM}{R^2}}$$

Substituting the values of $M = 6.4 \times 10^{24}$ kg and $R = 6.4 \times 10^6$ m, we get $g = 9.8 \text{ m/s}^2$.

Note

- The value of g on the surface of the moon is equal to 1/6 times the value of g on the surface of the earth.
- The value of g is independent of mass, shape, size etc., of the body and depends upon the mass and radius of the earth.

EXAMPLE |1| Mass of the Moon

The acceleration due to gravity at the moon's surface is 1.67 ms^{-2}. If the radius of the moon is 1.74×10^6 m, then calculate the mass of the moon.

Sol. $g = \frac{GM}{R^2}$ or $M = \frac{gR^2}{G}$

This relation is true not only to the earth but for any heavenly body which is assumed to be spherical.

Now, $g = 1.67 \text{ ms}^{-2}$, $R = 1.74 \times 10^6$ m

$G = 6.67 \times 10^{-11} \text{ Nm}^{-2} \text{ kg}^{-2}$

$$\therefore \text{ Mass of the moon, } M = \frac{1.67 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}} \text{ kg}$$

$$= 7.58 \times 10^{22} \text{ kg}$$

EXAMPLE |2| Star Heavier than Sun

A star 2.5 times the mass of the sun and collapsed to a size of 12 km and rotates with a speed of 1.2 revolutions per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects, called pulsars, belong to this category.) Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun $= 2 \times 10^{30}$ kg) **[NCERT]**

Sol. We are given mass of neutron star,

i.e. $M = 2.5 \times 2 \times 10^{30}$ kg

$= 5 \times 10^{30}$ kg

Radius of star, $R = 12 \text{ km} = 1.2 \times 10^4$ m

Frequency of rotation, $\nu = 1.5$

If g is the acceleration due to gravity on the surface of the star,

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 5 \times 10^{20}}{(1.2 \times 10^4)^2} \text{ m/s}^2$$

$$= 2.3 \times 10^{12} \text{ m/s}^2$$

Centrifugal acceleration (a_c) produced in the object at the equator,

i.e. $$a_c = R\omega^2$$

$$= R \times (2\pi\nu)^2 = 4\pi^2\nu^2 R$$

or $$a_c = 4 \times 9.87 \times (1.5)^2 \times 1.2 \times 10^4 \text{ m/s}^2$$

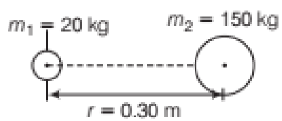
$$a_c = 1.1 \times 10^6 \text{ m/s}^2$$

Since $g > a_c$, the object will remain stuck to its surface due to gravity.

EXAMPLE |3| Mass of the Earth

A spherical mass of 20 kg lying on the earth's surface is attracted by another spherical mass of 150 kg with a force equal to the weight of 0.25 mg. The centre of the two masses are 0.30 m apart. Calculate the mass of the earth.

Sol.



Given, $m_1 = 20$ kg, $m_2 = 150$ kg

$F = 0.25$ mg-wt

$g = 9.80 \text{ ms}^{-2}$

$R = 6 \times 10^6$ m

Now, kg-wt multiply by 9.8 to get in Newton.

$$F = 0.25 \text{ mg-wt}$$

$$= 0.25 \times 10^{-3} \text{ g-wt}$$

$$= 0.25 \times 10^{-3} \times 10^{-3} \text{ kg-wt}$$

$$= 0.25 \times 10^{-6} \text{ kg-wt}$$

$$= 0.25 \times 10^{-6} \times 9.8 \text{ N} \quad [\because 1 \text{ kg-wt} = 9.8 \text{ N}]$$

From universal law, $F = G\dfrac{m_1 m_2}{r^2}$

or $$G = \frac{Fr^2}{m_1 m_2}$$

$$G = \frac{0.25 \times 10^{-6} \times 9.8 \times (0.3)^2}{20 \times 150}$$

$$G = 75 \times 9.80 \times 10^{-13} \text{ Nm}^2 \text{ kg}^{-2}$$

$$\therefore \quad g = \frac{GM}{R^2}$$

$$\therefore \quad M = \frac{g \times R^2}{G} = \frac{9.80 \times (6 \times 10^6)^2}{75 \times 9.80 \times 10^{-13}}$$

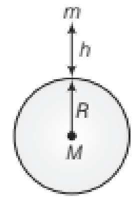
$$= 4.8 \times 10^{24} \text{ kg}$$

Acceleration due to Gravity above the Surface of Earth

Consider a body of mass m lying on the surface of the earth of mass M and radius R. Acceleration due to gravity at the surface of the earth.

i.e. $$g = \frac{GM}{R^2} \quad \text{...(i)}$$

Suppose the body is taken to a height h above the surface of the earth where the value of acceleration due to gravity is g_h as shown in figure.



Variation of g with altitude

Then, $$g_h = \frac{GM}{(R+h)^2} \quad \text{...(ii)}$$

where, $(R+h)$ is the distance between the centres of body and the earth.

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

or $$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} \quad \text{...(iii)}$$

As $h << R$, we can derive an expression for g_h

$$\frac{g_h}{g} = \frac{R^2}{\left[R\left(1+\frac{h}{R}\right)\right]^2} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

or $$\frac{g_h}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

$$\boxed{\text{Acceleration due to gravity at height } h,\ g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}}$$

If $h << R$, therefore h/R is very small, then expanding the right hand side of the above equation by Binomial theorem and neglecting squares and higher power of h/R, we get

$$\frac{g_h}{g} = 1 - \frac{2h}{R}$$

$$\boxed{\text{Acceleration due to gravity at height } h,\ g_h = g\left(1-\frac{2h}{R}\right)} \quad \text{...(iv)}$$

or $$gh = g - \frac{2h}{R}g$$

or $$g - g_h = \frac{2h}{R}g \quad \text{...(v)}$$

From Eq. (v), we know that the value of acceleration due to gravity decreases with height.

Since, the value of g at a given place on the earth is constant and R is also constant,

$$\therefore \qquad g - g_h \propto h$$

From the above equation, we note that if h increases, g_h must decrease because g is constant. **Thus, the value of acceleration due to gravity decreases with increase in height above the surface of the earth.**

With height h, the decrease in the value of g is

$$\Rightarrow \qquad g - g_h = \frac{2hg}{R}$$

$\therefore$ Fractional decrease in the value of g is

$$\Rightarrow \qquad \frac{g - g_h}{g} = \frac{2h}{R}$$

$\therefore$ % decrease in the value of g

$$= \left(\frac{g - g_h}{g}\right) \times 100 = \frac{2h}{R} \times 100\%$$

EXAMPLE |4| Gravitational Pull on a Body

A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth? **[NCERT]**

Sol. Let g_h be the acceleration due to gravity at a height equal to half the radius of the earth $\left(h = \frac{R}{2}\right)$ and g is its value on the earth's surface. Let the body have mass m. We know that

$$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 \quad \text{or} \quad \frac{g_h}{g} = \left(\frac{R}{R + \frac{R}{2}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Let w be the weight of body on the surface of the earth and w_h be the weight of the body at height h.

$$\text{Then,} \qquad \frac{w_h}{w} = \frac{mg_h}{mg} = \frac{g_h}{g} = \frac{4}{9}$$

$$\text{or} \qquad w_h = \frac{4}{9} w = \frac{4}{9} \times 63 \text{ N} = 28 \text{ N}$$

EXAMLE |5| Free Fall Acceleration From Space

If an object at the altitude of the space shuttle's orbit, about 400 km about the earth's surface, then find out the free fall acceleration of that object.

Sol. The acceleration, $g = \frac{F}{m} = \frac{GMm/R^2}{m} = \frac{GM}{R^2}$

If the object is at height h above the earth's surface, then

$$g_h = \frac{GM}{(R+h)^2}$$

$$\Rightarrow \qquad g_h = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.4 \times 10^6 + 0.4 \times 10^6)^2} = 8.70 \text{ m/s}^2$$

Acceleration due to Gravity below the Earth's Surface

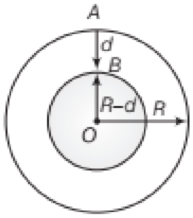
Assume the earth to be a homogeneous sphere (i.e. a sphere of uniform density). Let ρ be the mean density of the earth and a body be lying on the surface of the earth where the value of acceleration due to gravity is g.

$$\text{Then,} \qquad g = \frac{GM}{R^2} \quad \text{or} \quad g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$\left[\because \text{ mass of the earth, } M = \frac{4}{3}\pi R^3 \rho\right]$$

$$g = \frac{4}{3}\pi G R \rho \qquad \ldots\text{(i)}$$

Now, the body be taken to a depth d below the free surface of the earth, where the acceleration due to gravity is g_d. Here, the force of gravity acting on the body is only due to the inner solid sphere of radius $(R - d)$.



Variation of g with depth

$$\therefore \qquad g_d = \frac{GM'}{(R-d)^2}$$

where M' is the mass of inner solid sphere of radius $(R - d)$.

$$\text{or} \qquad g_d = \frac{G}{(R-d)^2} \times \frac{4}{3}\pi (R-d)^3 \times \rho$$

$$\text{or} \qquad g_d = \frac{4}{3}\pi G (R-d)\rho \qquad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{g_d}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi G R \rho} = \frac{R-d}{R}$$

$$\text{or} \qquad \frac{g_d}{g} = 1 - \frac{d}{R}$$

$$\boxed{\text{Acceleration due to gravity at depth } d, \; g_d = g\left(1 - \frac{d}{R}\right)}$$

$$\text{or} \qquad g_d - g = -\frac{d}{R_,} g$$

$$\text{or} \qquad g - g_d = \frac{d}{R} g$$

Here, $(g - g_d)$ gives the decrease in the value of g.

Since, g is constant at a given place of the earth and R is also a constant.

$$\therefore \qquad (g - g_d) \propto d$$

From the above equation, it is clear that if d increases, g_d must decrease because g is constant. **Hence, the acceleration due to gravity decreases as we move down into the surface of the earth**. i.e. g decreases with the increase in depth d.

At the centre O of the earth, $d = R$

Thus, we get $\qquad g_d = g\left(1 - \frac{R}{R}\right) = \text{zero}$

Hence, acceleration due to gravity at the centre of the earth is zero.

Hence, the weight of a body at the centre of the earth is zero though its mass is constant.

Decrease in the value of g with depth d is

$$g - g_d = \frac{g_d}{R}$$

$\therefore$ Fractional decrease in the value of g with depth

$$= \frac{g - g_d}{g} = \frac{d}{R}$$

$\therefore$ % decrease in the value of g

$$= \frac{g - g_d}{g} \times 100 = \frac{d}{R} \times 100\%$$

EXAMPLE |6| Variation in g at the Sea Level

What will be the value of g at the bottom of sea 7 km deep? Diameter of the earth is 12800 km and g on the surface of the earth is 9.8 ms^{-2}.

Sol. Depth of sea, $d = 7$ km, $g = 9.8\ ms^{-2}$

Radius of the earth, $R = \frac{D}{2} = \frac{12800}{2}$ km $= 6400$ km

Value of g at bottom of sea

$$g_d = g\left(1 - \frac{d}{R}\right)$$

$$= 9.8\left(1 - \frac{7}{6400}\right) ms^{-2}$$

$$g_d = \frac{9.8 \times 6393}{6400} ms^{-2}$$

$$= 9.789\ ms^{-2}$$

Note

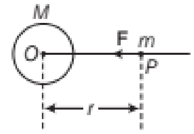
Acceleration due to gravity also vary due to the rotation of earth, if a body of mass m lying at a point whose latitude is λ, then rotation of earth (angular speed ω), the apparent acceleration due to gravity on body is given by $g' = g - \omega^2 R\cos^2\lambda$

INTENSITY OF GRAVITATIONAL FIELD AT A POINT

It is the force experienced by a unit mass placed at that point.

Let, M be the mass of a body around which a gravitational field exists. In order to get the gravitational field intensity at a point P in the gravitational field, a test mass m is placed at the point P.

The test mass is supposed to be so small that it does not alter the gravitational field in any manner. If a test mass m at a point P in a gravitational field experience a force F, then



Gravitational field intensity

Intensity of gravitational field, $I = \frac{F}{m} = \frac{GMm/r^2}{m}$

$$\boxed{\text{Intensity of gravitational field, } I = \frac{GM}{r^2}}$$

where, r is the distance of P from the centre of the body producing the gravitation field.

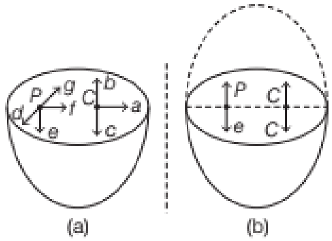
If $r = \infty$, $I = 0$. It means the intensity of gravitational field is zero only at infinite distance from the body.

If a unit mass (m) is placed on the surface of earth, then the gravitational force acting on the test mass m will be equal to the weight w of the test mass.

Now, $\qquad I = \frac{w}{m} = \frac{mg}{m} = g \quad$ or $\quad I = g$

EXAMPLE |7| A Hemispherical Shell

The gravitational intensity at the centre of a hemispherical shell of uniform mass density, (in figure) has the direction indicated by arrow (i) a, (ii) b, (iii) c, (iv) zero. Choose the correct answer. **[NCERT]**



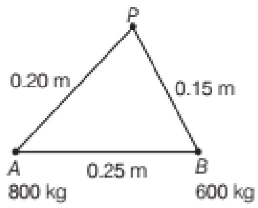
Hemispherical shell

Sol. Complete the hemisphere, Fig. (b). The gravitational intensity (gravitational force per unit mass) is zero at all points in a spherical shell. This implies that at any point inside the spherical shell, the gravitational forces are symmetrically placed. As gravitational potential v is constant, hence intensity, $E = -\frac{dv}{dr} = 0$ at C.

Thus, if the upper hemisphere is removed, the net gravitational force acting on the particle at C will be downward as shown by arrow c.
Hence, option (iii) is correct. It is to be remembered that the net gravitational force acting on particle C due to upper hemisphere (shown by dotted arrow), is equal and opposite to that acting due to lower hemisphere in order that the total gravitational force due to entire spherical shell is zero.

EXAMPLE |8| Gravitational Field Intensity
Two masses 800 kg and 600 kg, are at a distance 0.25 m apart. Calculate the magnitude of the gravitational field intensity at a point distance 0.20 m from the 800 kg mass and 0.15 m from the 600 kg mass.
Given, $G = 6.6 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$.

Sol. Let I_A be the gravitational field intensity at P due to 800 kg mass at A.



Then, $I_A = G\frac{800}{(0.2)^2}$

$= 2 \times 10^4$ along PA

Let I_B be the gravitational field intensity at P due to 600 kg mass at B

Then, $$I_B = G\frac{600}{(0.15)^2} \text{ along } PB$$

$$= \frac{80000\,G}{3} \text{ along } PB$$

The angle between I_A and I_B is 90°.
If I be the magnitude of the resultant intensity, then

$$I = \sqrt{I_A^2 + I_B^2}$$

$$= \sqrt{(2 \times 10^4\,G)^2 + \left(\frac{80000\,G}{3}\right)^2}$$

$$= G\sqrt{4 \times 10^8 + \frac{64}{9} \times 10^8}$$

$$= 6.67 \times 10^{-11} \times 2 \times 10^4 \sqrt{1 + \frac{16}{9}}$$

$$= 6.67 \times 2 \times \frac{5}{3} \times 10^{-7}\ \text{Nkg}^{-1}$$

$$I = 2.22 \times 10^{-6}\ \text{Nkg}^{-1}$$

GRAVITATIONAL POTENTIAL

Gravitational potential at a point in the gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration

i.e. $$V = -\frac{W}{m} = -\int\frac{\mathbf{F}\cdot\mathbf{dr}}{m} = -\int\mathbf{I}\cdot\mathbf{dr} \qquad \left[\text{as, } \frac{F}{m} = I\right]$$

where, W is the amount of work done in bringing a body of mass m from infinity to that point and I is the intensity of field.

$$\therefore \qquad I = -\frac{dV}{dr}$$

i.e. negative gradient of potential gives intensity of field or potential which is a scalar function of position whose space derivative gives intensity.

Negative sign indicates that the direction of intensity is in the direction where the potential decreases. Gravitational potential is a scalar quantity and SI is joule/kg and in CGS system is erg/g.

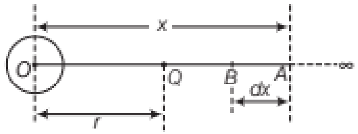
Dimensional formula for gravitational potential.

i.e. $$V = \frac{W}{m} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{M}]} = [\text{M}^0\text{L}^2\text{T}^{-2}]$$

Expression for Gravitational Potential

Suppose the earth be perfect sphere of radius R and mass M, can be supposed to be concentrated at its centre O. Then, the gravitational potential at point Q can be calculated, where $OQ = r$ and $r > R$.

Take two points A and B, so $OA = x$ and $AB = dx$ as shown in figure.



Calculation for gravitational potential

At point A, gravitational force of attraction on a body of unit mass will be $$F = \frac{GM \times 1}{x^2} = \frac{GM}{x^2}$$

Small amount of work done in bringing the unit mass body without acceleration through a small distance ($AB = dx$) is

$$dW = Fdx = \frac{GM}{x^2}dx$$

Total amount of work done in bringing a body from infinity to point Q, we get

$$\text{Work done } (W) = \int_{\infty}^{r} \frac{GM}{x^2}\,dx = -\left[\frac{GM}{x}\right]_{\infty}^{r}$$

$$= -GM\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{-GM}{r}$$

$\because$ $\quad V = W$

$$\boxed{\text{Gravitational potential, } V = \frac{-GM}{r}} \quad \text{...(i)}$$

where, V is total gravitational potential at point Q.

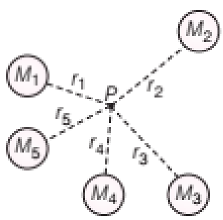
Special Cases

(i) When $r = \infty$ from Eq. (i), then $V_Q = 0$, hence gravitational potential is maximum (zero) at infinity.

(ii) At surface of the earth, $r = R$, then $V_Q = \frac{-GM}{R}$.

Note

Potential due to a large number of particles is given by scalar addition of all the potentials as shown in figure.



Superposition of different potentials

$$V = V_1 + V_2 + V_3 + \dots$$
$$= \frac{-GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{GM_3}{r_3} \dots = -G\sum_{i=1}^{i=n}\frac{M_i}{r_i}$$

EXAMPLE |9| Potential on Geostationary Satellite

As you will learn in the text, a geostationary satellite orbits the earth at a height of nearly 36000 km from the surface of the earth. What is the potential due to the earth's gravity at the site of this satellite? (take the potential energy at infinity to be zero). Mass of the earth $= 6.0 \times 10^{24}$ kg, radius = 6400 km. **[NCERT]**

Sol. We are given that

Mass of the earth, $M = 6.0 \times 10^{24}$ kg

Radius of the earth, $R = 6400$ km

Height of the satellite from the earth's surface,

$h = 36000$ km

Distance of the satellite from the centre of the earth,

$r = R + h = 6400 \text{ km} + 36000 \text{ km}$

$r = 42400 \text{ km} = 4.24 \times 10^7$ m

If V is the potential at the site of the satellite,

$$V = -\frac{GM}{r} = -\frac{(6.67\times10^{-11})\times(6.0\times10^{24})}{(4.24\times10^{7})}$$

$V = -9.4 \times 10^6$ J/kg

EXAMPLE |10| Dependence of Gravity on Altitude

At a point above the surface of the earth, the gravitational potential is -5.12×10^7 J/kg and the acceleration due to gravity is 6.4 m/s^2. Assuming the mean radius of the earth to be 6400 km, calculate the height of the point above the earth's surface.

Sol. If r is the distance of the given point from the centre of the earth, then gravitational potential at the point,

$$V = -\frac{GM}{r} = -5.12 \times 10^7 \text{ J/kg}$$

Acceleration due to gravity at this point,

$$g = \frac{GM}{r^2} = 6.4 \text{ m/s}^2$$

Clearly, $\quad \frac{|V|}{g} = \frac{GM/r}{GM/r^2} = r$

Thus, $\quad r = \frac{5.12\times10^7 \text{ J/kg}}{6.4 \text{ m/s}^2} = 8\times10^6 \text{ m} = 8000 \text{ km}$

Obviously, height of the point from the earth's surface

$= (r - R) = 8000 \text{ km} - 6400 \text{ km} = 1600 \text{ km}$

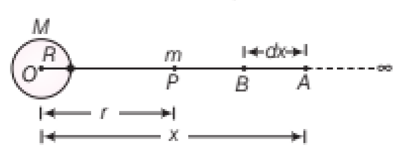
GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of a body at a point is defined as the amount of work done in bringing the given body from infinity to that point against the gravitational force.

Let a body of mass m is placed at P in the gravitational field of a body of mass M.

Let r be the distance of P from the centre O of the body of mass M as shown in figure. Then, the total work done (W) by the gravitational field when a body of mass m is moved from infinity to a distance r from O is given by

$$W = -\frac{GmM}{r}$$



Gravitational potential energy

This work done is equal to the gravitational potential energy U of mass m.

$\therefore$ Gravitational potential energy, $U = -\dfrac{GMm}{r}$

Now, $$U = -\frac{GMm}{r}$$

$\Rightarrow$ $\boxed{\text{Gravitational potential energy, } U = \left(-\dfrac{GM}{r}\right) \times m}$

$\therefore$ **Gravitational potential energy = Gravitational potential $\times$ mass of the body.**

Some Cases

(i) According to the superposition principle, if there are three particles of masses m_1, m_2 and m_3 in an isolated system, then total gravitational potential energy of the system is

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + \frac{Gm_1m_3}{r_{13}}\right)$$

(ii) If the body of mass m is moved from the surface of the earth to a point distance h above the surface of the earth.

Then, change in potential energy or work done against gravity will be

$$W = \Delta U = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Change in gravitational potential energy at height h

$$\Delta U = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

[as, $r_1 = R$ and $r_2 = R + h$]

$$\Rightarrow \quad \Delta U = \frac{GMmh}{R^2\left(1+\dfrac{h}{R}\right)} = \frac{mgh}{1+\dfrac{h}{R}} \qquad \left(\text{as, } \frac{GM}{R^2} = g\right)$$

$$\Delta U = mgh\left(1+\frac{h}{R}\right)^{-1} = mgh\left(1-\frac{h}{R}\right)$$

$\because$ $h << R$

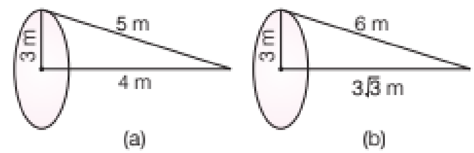
$\therefore$ $\Delta U = mgh$

(iii) When $h = R$, then work done $= \dfrac{mgR}{1+R/R} = \dfrac{1}{2}mgR$

EXAMPLE |11| Work Done by Uniform Axial Ring

A particle of mass 1 kg is placed at a distance of 4 m from the centre and on the axis of a uniform ring of mass 5 kg and radius 3 m. Calculate the work required to be done to increase the distance of the particle from 4 m to $3\sqrt{3}$ m.

Sol. $U_1 = -\dfrac{G \times 5 \times 1}{5}$ J or $U_1 = -G$ J



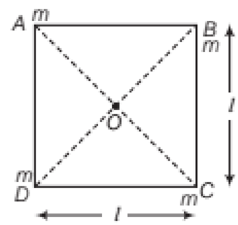
$$U_2 = \frac{-G \times 5 \times 1}{6}\text{J} = -\frac{5G}{6}\text{J}$$

$$\text{Work done} = \left(-\frac{5G}{6}\right) - (-G) = G - \frac{5G}{6} = \frac{G}{6}.$$

EXAMPLE |12| Potential Energy of a System of four Particles.

Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also, obtain the potential at the centre of the square. **[NCERT]**

Sol.



$$AB = BC = CD = DA = l$$

Here, $$AC = BD = \sqrt{l^2 + l^2} = \sqrt{2l^2} = l\sqrt{2}$$

$\therefore$ $$OA = OB = OC = OD = \frac{l\sqrt{2}}{2} = \frac{l}{\sqrt{2}}$$

Applying principle of superposition

Total potential energy of the system of particles is

$$U = U_{BA} + (U_{CB} + U_{CA}) + (U_{DA} + U_{DB} + U_{DC})$$

$$= 4U_{BA} + 2U_{DB}$$

$[\because U_{BA} = U_{DA} = U_{DC} = U_{CB}, U_{CA} = U_{DB}]$

$$= 4\left(\frac{-Gmm}{l}\right) + 2\left(\frac{-Gmm}{l\sqrt{2}}\right)$$

$$= \frac{-2Gm^2}{l}\left(2 + \frac{1}{\sqrt{2}}\right) = \frac{-2Gm^2}{l}(2 + 0.707)$$

$$U = \frac{-5.41}{l}Gm^2$$

Total gravitational potential at the centre O,

$$V = V_A + V_B + V_C + V_D$$

$$= 4V_A = 4\left(\frac{-Gm}{l_{OA}}\right) = 4 \times \left(\frac{-Gm}{l/\sqrt{2}}\right)$$

$$V = \frac{-4\sqrt{2}\,Gm}{l}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity **[NCERT Exemplar]**

(a) will be directed towards the centre but not the same everywhere
(b) will have the same value everywhere but not directed towards the centre
(c) will be same everywhere in magnitude directed towards the centre
(d) cannot be zero at any point

Sol. (d) If we assume the earth as a sphere of uniform density, then it can be treated as point mass placed at its centre. In this case acceleration due to gravity $g = 0$, at the centre.

It is not so, if the earth is considered as a sphere of non-uniform density, in that case value of g will be different at different points and cannot be zero at any point.

2. If g is the acceleration due to gravity at the surface of the earth. The force acting on the particle of mass m placed at the surface is

(a) mg (b) $\frac{GmM_e}{R_e^2}$
(c) Data insufficient (d) Both (a) and (b)

Sol. (d) Force on particle at surface is

$$F = mg$$

where, g = acceleration due to gravity at the earth's surface

Also, $$g = \frac{GM_e}{R_e^2}$$

$\Rightarrow$ $$F = mg = \frac{GmM_e}{R_e^2}$$

3. Earth is flattened at the poles and bulges at the equator. This is due to the fact that

(a) the earth revolves around the sun in an elliptical orbit
(b) the angular velocity of spinning about its axis is more at the equator
(c) the centrifugal force is more at the equator than at poles
(d) None of the above

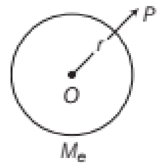
Sol. (c) Higher centrifugal force causes bulging of earth at equator.

4. The gravitational potential at a distance r from the centre of the earth $(r > R)$ is given by (consider, mass of the earth $= M_e$, radius of the earth $= R$)

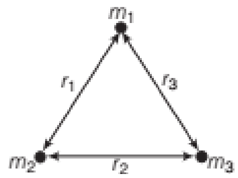
(a) $\frac{-GM_e}{R}$ (b) $\frac{GM_e}{R}$ (c) $\frac{-GM_e}{r}$ (d) $\frac{+GM_e}{r}$

Sol. (c) The gravitational potential at a distance r from the centre of the earth.

$$V(r) = -\frac{GM_e}{r}$$



5. Gravitational potential energy of a system of particles as shown in the figure is



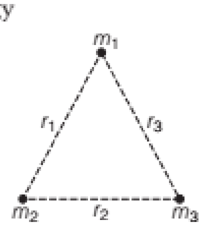
(a) $\frac{Gm_1m_2}{r_1} + \frac{Gm_2m_3}{r_3} + \frac{Gm_1m_3}{r_3}$

(b) $\left(\frac{-Gm_1m_2}{r_1}\right) + \left(\frac{-Gm_2m_3}{r_2}\right) + \left(\frac{-Gm_1m_3}{r_3}\right)$

(c) $\frac{-Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} + \frac{Gm_1m_3}{r_3}$

(d) $\frac{Gm_1m_2}{r_1} + \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$

Sol. (b) For a system of particles, all possible pairs are taken and total gravitational potential energy is the algebraic sum of the potential energies due to each pair, applying the principle of superposition. Total gravitational potential energy



$$= \frac{-Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$$

$$= \left(\frac{-Gm_1m_2}{r_1}\right) + \left(\frac{-Gm_2m_3}{r_2}\right) + \left(\frac{-Gm_1m_3}{r_3}\right)$$

VERY SHORT ANSWER Type Questions

6. Where does a body weigh more; at the surface of the earth or in a mine?

Sol. The value of g in mine is less than that on the surface of the earth.

7. If the earth is regarded as a hollow sphere, then what is the weight of an object below surface of the earth?

Sol. The weight of an object below surface of the earth is zero.

8. What is the amount of work done in bringing a mass from the surface of the earth on one side to a point diametrically opposite on the other side?

Sol. Since, gravitational potential difference is zero, therefore the work done is zero.

9. If the force of gravity acts on all bodies in proportion to their masses, then why doesn't a heavy body fall faster than a light body?

Sol. Acceleration due to gravity is independent of the mass of the body.

10. What would happen if the force of gravity were to disappear suddenly?

Sol. The universe would collapse. We would be thrown away because of the centrifugal force. Eating, drinking and infact all activities would become impossible.

11. When a pendulum clock is taken to a mountain, it becomes slow. But a wrist watch controlled by a spring remains unaffected. Explain.

Sol. At the mountain, g decreases and time period of the pendulum clock increases at $T = 2\pi\sqrt{l/g}$. On the other hand, a spring in the wrist watch remains unaffected by the variation of g.

12. What is the apparent weight of a man of 60 kg who is standing in a lift which is moving up with a uniform speed?

Sol. Apparent weight $= mg = 60 \times 10\ \text{N} = 600\ \text{N}$

13. Would we have more sugar to the kilogram at the pole or at the equator?

Sol. $mg_p = m' g_e$ since, $g_p > g_e$,

$\therefore$ $m' > m$. So, we shall have greater mass of sugar at the equator.

14. Why a body weighs more at poles and less at equator?

Sol. The value of g is more at poles than at the equator. Therefore, a body weighs more at poles than at equator.

15. Give a method for the determination of the mass of the moon.

Sol. By making use of the relation,

$$g_m = \frac{GM_m}{R_m^2}$$

16. Define the effect of shape of the earth on the value of g.

Sol. At the surface of the earth, g is maximum at the poles and minimum at the equator i.e. the value of g decreases from poles to equator.

SHORT ANSWER Type Questions

17. Why a tennis ball bounces higher on hills than on plains?

Sol. As the acceleration due to gravity on hills is less than that on the surface of the earth (effect of height), therefore, a tennis ball bounces higher on hills than on plains.

18. The acceleration due to gravity on a planet is $1.96\ \text{ms}^{-2}$. If it is safe to jump from a height of 2 m on the earth, then what will be the corresponding safe height on the planet?

Sol. The safety of a person depends upon the momentum with which the person hits the planet. Since, the mass of the person is constant, therefore the maximum velocity v is the limiting factor.

19. Does the concentration of the earth's mass near its centre change the variation of g with height compared with a homogeneous sphere, how?

Sol. Any change in the distribution of the earth's mass will not affect the variation of acceleration due to gravity with height. This is because for a point outside the earth, the whole mass of the earth is effective and the earth behaves as a homogeneous sphere.

20. Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh $\frac{3}{5}$th as much as at present. Take the equatorial radius as 6400 km.

Sol. Acceleration due to gravity at the equator is

$$g_e = g - R\omega^2$$

$$mg_e = mg - mR\omega^2$$

or $$\frac{3}{5}mg = mg - mR\omega^2 \qquad \left[\because mg_e = \frac{3}{5}mg\right]$$

$$\therefore \quad \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2\times 9.8}{5\times 6400\times 10^3}} = 7.8\times 10^{-4}\ \text{rad/s}$$

21. What is the gravitational potential energy of a body at height h from the earth surface?

Sol. Gravitational potential energy, i.e.

$$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h} \quad \left[\text{where, } g = \frac{GM}{R^2}\right]$$

$$= -\frac{gR^2m}{R\left(1+\frac{h}{R}\right)} = -\frac{mgR}{1+\frac{h}{R}}$$

22. A spherical planet has mass M_P and diameter D_P. A particle of mass m falling freely near the surface of this planet will **experience** an acceleration due to gravity, equal to whom?

Sol. Force is given by

$$F = \frac{GM_em}{R^2} = \frac{GM_pm}{(D_p/2)^2} = \frac{4GM_pm}{D_p^{\,2}}$$

$$\frac{F}{m} = \frac{4GM_p}{D_p^{\,2}}$$

LONG ANSWER Type I Questions

23. Assuming the earth to be a sphere of uniform mass density, how much would body weigh half way down to the centre of the earth if it weighted 250 N on the surface? **[NCERT]**

To calculate the weight of the body down the centre of the earth, we have to calculate the acceleration due to gravity at depth d from earth's surface which is given by

$$g' = g\left(1-\frac{d}{R}\right)$$

Sol. Weight of the body at the earth's surface

$$w = mg = 250 \text{ N} \qquad \text{...(i)}$$

Acceleration due to gravity at depth d from the earth's surface

$$g' = g\left(1-\frac{d}{R}\right)$$

Here, $d = \frac{R}{2}$

$\therefore$ $g' = g\left(1-\frac{R/2}{R}\right) = g\left(1-\frac{1}{2}\right)$

$\Rightarrow$ $g' = \frac{g}{2}$

$\therefore$ Weight of the body at depth d

$\Rightarrow$ $w' = mg = \frac{mg}{2}$

Using Eq. (i), we get

$$w' = \frac{250}{2} = 125 \text{ N}$$

$\therefore$ Weight of the body will be 125 N.

24. An object of mass m is raised from the surface of the earth to a height equal to the radius of the earth, that is, taken from a distance R to $2R$ from the centre of the earth. What is the gain in its potential energy? **[NCERT Exemplar]**

Sol. Gain in PE, $\Delta U = U_f - U_i$

$$= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMm}{R}\left(-\frac{1}{2}+1\right) = \frac{GMm}{2R}$$

$$= \frac{(gR^2)\,m}{2R} = \frac{1}{2}mgR \qquad \left[\text{as } g = \frac{GM}{R^2}\right]$$

25. Derive an expression for work done against gravity.

Sol. Potential energy of the body on the surface of the earth $= \frac{-GMm}{R}$

Potential energy of the body at a height h from the surface of the earth $= -\frac{GMm}{(R+h)}$

$$\text{Work done} = \left(-\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMm}{R} - \frac{GMm}{R+h}$$

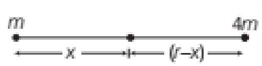
$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= \frac{GMmh}{R(R+h)} = \frac{MgR^2h}{R(R+h)} \qquad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{(Mgh)R}{(R+h)} = \frac{Mgh}{1+\frac{h}{R}}$$

26. Two bodies of m and $4m$ are placed at a distance r. The gravitational field is zero at a point on the line joining the two masses. What will be the gravitational potential at this point? **[AIEEE 2011]**

Sol.



$$\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$$\frac{1}{x} = \frac{2}{r-x}$$

$\Rightarrow$ $r - x = 2x$

$$\Rightarrow \qquad 3x = r \Rightarrow x = \frac{r}{3}$$

$$\therefore \text{ The gravitational potential} = \frac{-Gm}{r/3} \frac{-G(4m)}{2r/3}$$

$$= \frac{-3Gm}{r} \frac{-6Gm}{r} = \frac{-9Gm}{r}$$

27. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km, respectively.

What is the required energy for this work done?

Sol. $W = 0 - \left[\frac{-GMm}{R}\right] = \frac{GMm}{R}$

$$= gR^2 \times \frac{m}{R} = mgR$$

$$= 1000 \times 10 \times 6400 \times 10^3$$

$$= 64 \times 10^9 \text{ J} = 6.4 \times 10^{10} \text{ J}$$

28. What will be the potential energy of a body of mass 67 kg at a distance of 6.6×10^{10} m from the centre of the earth? Find gravitational potential at this distance.

Sol. Mass of the earth, $M = 6.0 \times 10^{24}$ kg, $m = 67$ kg

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

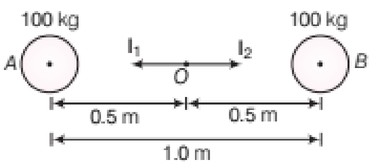
Gravitational potential, $V = -\frac{GM}{R}$

$$= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.6 \times 10^{10}}$$

$$V = -6.1 \times 10^3 \text{ Jkg}^{-1}$$

29. Two heavy spheres, each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid-point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable? **[NCERT]**

Sol. Let A and B be the two given spheres and O be the mid-point on the line joining their centres.



If I_1 and I_2 are the gravitational fields at O due to A and B respectively,

$$I_1 = \frac{G \times 100}{(0.5)^2}, \quad I_2 = \frac{G \times 100}{(0.5)^2}$$

Since, I_1 and I_2 are equal and opposite, then resultant gravitational field at O is zero.

If V is the gravitational potential at O due to A and B

$$V = -\frac{G \times 100}{0.5} - \frac{G \times 100}{0.5}$$

$$= -\frac{2G \times 100}{0.5}$$

or $$V = \frac{-2 \times 6.67 \times 10^{-11} \times 100}{0.5}$$

$$= -2.7 \times 10^{-8} \text{ J/kg}$$

The object placed at O is in equilibrium as there is no net force acting on the object. But it will be in an unstable equilibrium as once displaced from O, it will not come back to O.

LONG ANSWER Type II Questions

30. Choose the correct alternatives.

(i) Acceleration due to gravity increases/ decreases with increasing altitude.

(ii) Acceleration due to gravity increases/ decreases with increasing depth (assume the earth to be a sphere of uniform density).

(iii) Acceleration due to gravity is independent of the mass of the earth/mass of the body.

(iv) The formula, $-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth. **[NCERT]**

Sol. (i) Acceleration due to gravity at altitude h from the earth's surface is given by

$$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

where, R_e is the radius of the earth.

Therefore, acceleration due to gravity decreases with increasing altitude.

(ii) Acceleration due to gravity at depth d from the earth's surface is given by

$$g' = g\left(1 - \frac{d}{R_e}\right)$$

Therefore, acceleration due to gravity decreases with increasing depth.

(iii) Acceleration due to gravity is independent of the mass of the body.

(iv) The formula, $-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

is more accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1, distance away from the centre of the earth.

31. Two stars each of 1 solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head on collision. When they are at a distance of 10^9 km, their speeds are negligible. What is the speed with which they will collide? The radius of each star is 10^4 km. Assume the stars to remain undistored until they collide (use the known value of G). **[NCERT]**

Sol. Here, mass of each star, $M = 2 \times 10^{30}$ kg

Radius of each star, $r = 10^7$ m

Initial potential energy of the stars when they are 10^{12} m apart $= -\dfrac{GM \times M}{10^{12}} = -\dfrac{GM^2}{10^{12}}$

[distance between two stars $= 10^{12}$ m]

When the stars are just going to collide, the distance between their centres = twice the radius of each star $= 2r = 2 \times 10^7$ m

Final potential energy of the stars when they are about to collide $= -G\dfrac{M \times M}{2 \times 10^7} = -\dfrac{GM^2}{2 \times 10^7}$

Change in potential energy of stars

$$= -\frac{GM^2}{10^{12}} - \left(-\frac{GM^2}{2 \times 10^7}\right) = \frac{GM^2}{2 \times 10^7} - \frac{GM^2}{10^{12}}$$

$$= \frac{GM^2}{2 \times 10^7} \left[\text{as } \frac{GM^2}{10^{12}} << \frac{GM^2}{2 \times 10^7}\right] \quad \text{...(i)}$$

Let v be the speed of each star just before colliding.

Final KE of the stars $= 2 \times \dfrac{1}{2} Mv^2 = Mv^2$

Initial KE of the stars = 0

(because when the stars are initially 10^{12} m apart, their speeds are negligible).

Change in KE of the stars $= Mv^2$ ...(ii)

Using the law of conservation of energy, from Eqs. (i) and (ii), we get $\dfrac{GM^2}{2 \times 10^7} = Mv^2$

or $v = \sqrt{\dfrac{GM}{2 \times 10^7}}$ or $v = \sqrt{\dfrac{6.67 \times 10^{-11} \times (2 \times 10^{30})}{2 \times 10^7}}$

or $v = 2.6 \times 10^6$ m/s

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. What is the value of acceleration caused by force of gravity on a stone placed on ground?

(a) 10 ms $^{-2}$ (b) 9.8 ms $^{-2}$

(c) 0 (d) - 9.81 ms $^{-2}$

2. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is

(a) $g_1 : g_2 = \dfrac{\rho_1}{R_1^2} : \dfrac{\rho_2}{R_2^2}$

(b) $g_1 : g_2 = R_1 R_2 : \rho_1 \rho_2$

(c) $g_1 : g_2 = R_1 \rho_2 : R_2 \rho_1$

(d) $g_1 : g_2 = R_1 \rho_1 : R_2 \rho_2$

3. The value of acceleration due to gravity at a height h above the surface of the earth of radius R is g', then

(a) $g' < g$ (b) $g' > g$

(c) $g' = g$ (d) $g' = g\left(1 - \dfrac{2h}{R}\right)$

4. The gravitational potential energy of a system consisting of two particles separated by a distance r is

(a) directly proportional to product of the masses of particles

(b) inversely proportional to the separation between them

(c) independent of distance r

(d) Both (a) and (b)

5. Two point masses m_1 and m_2 are separated by a distance r. The gravitational potential energy of the system is G_1. When the separation between the particles is doubled, the gravitational potential energy is G_2. Then, the ratio of $\dfrac{G_1}{G_2}$ is

(a) 1 (b) 2 (c) 3 (d) 4

Answer

1. (c) | 2. (d) | 3. (a) | 4. (d) | 5. (b)

31. Two stars each of 1 solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head on collision. When they are at a distance of 10^9 km, their speeds are negligible. What is the speed with which they will collide? The radius of each star is 10^4 km. Assume the stars to remain undistored until they collide (use the known value of G). **[NCERT]**

Sol. Here, mass of each star, $M = 2 \times 10^{30}$ kg

Radius of each star, $r = 10^7$ m

Initial potential energy of the stars when they are 10^{12} m apart $= -\frac{GM \times M}{10^{12}} = -\frac{GM^2}{10^{12}}$

[distance between two stars $= 10^{12}$ m]

When the stars are just going to collide, the distance between their centres = twice the radius of each star $= 2r = 2 \times 10^7$ m

Final potential energy of the stars when they are about to collide $= -G\frac{M \times M}{2 \times 10^7} = -\frac{GM^2}{2 \times 10^7}$

Change in potential energy of stars

$$= -\frac{GM^2}{10^{12}} - \left(-\frac{GM^2}{2 \times 10^7}\right) = \frac{GM^2}{2 \times 10^7} - \frac{GM^2}{10^{12}}$$

$$\approx \frac{GM^2}{2 \times 10^7} \left[\text{as } \frac{GM^2}{10^{12}} << \frac{GM^2}{2 \times 10^7}\right] \quad \text{...(i)}$$

Let v be the speed of each star just before colliding.

Final KE of the stars $= 2 \times \frac{1}{2} Mv^2 = Mv^2$

Initial KE of the stars = 0

(because when the stars are initially 10^{12} m apart, their speeds are negligible).

Change in KE of the stars $= Mv^2$ ...(ii)

Using the law of conservation of energy, from Eqs. (i) and (ii), we get $\frac{GM^2}{2 \times 10^7} = Mv^2$

or $v = \sqrt{\frac{GM}{2 \times 10^7}}$ or $v = \sqrt{\frac{6.67 \times 10^{-11} \times (2 \times 10^{30})}{2 \times 10^7}}$

or $v = 2.6 \times 10^6$ m/s

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. What is the value of acceleration caused by force of gravity on a stone placed on ground?

(a) 10 ms $^{-2}$ (b) 9.8 ms $^{-2}$

(c) 0 (d) ~ 9.81 ms $^{-2}$

2. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is

(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$

(b) $g_1 : g_2 = R_1 R_2 : \rho_1 \rho_2$

(c) $g_1 : g_2 = R_1 \rho_2 : R_2 \rho_1$

(d) $g_1 : g_2 = R_1 \rho_1 : R_2 \rho_2$

3. The value of acceleration due to gravity at a height h above the surface of the earth of radius R is g', then

(a) $g' < g$ (b) $g' > g$

(c) $g' = g$ (d) $g' = g\left(1 - \frac{2h}{R}\right)$

4. The gravitational potential energy of a system consisting of two particles separated by a distance r is

(a) directly proportional to product of the masses of particles

(b) inversely proportional to the separation between them

(c) independent of distance r

(d) Both (a) and (b)

5. Two point masses m_1 and m_2 are separated by a distance r. The gravitational potential energy of the system is G_1. When the separation between the particles is doubled, the gravitational potential energy is G_2. Then, the ratio of $\frac{G_1}{G_2}$ is

(a) 1 (b) 2 (c) 3 (d) 4

Answer

1. (c)	2. (d)	3. (a)	4. (d)	5. (b)

VERY SHORT ANSWER Type Questions

6. If the radius of the earth is 6000 km, then what will be the weight of 120 kg body if taken to a height of 2000 km above sea level? [**Ans.** -67.5 kg-wt]

7. What is the value of gravitational potential at the surface of the earth, referred to zero potential at infinite distance? [**Ans.** -6.25×10^{7} J / kg]

8. If the radius of the earth were increased by a factor of 3, then by what factor would its density have to be changed to keep g the same? [**Ans.** $\rho/3$]

9. The particles of masses 0.2 kg and 0.8 kg are separated by 12 cm. At which point from the 0.2 kg particle, the gravitational field intensity due to the two particles is zero? [**Ans.** $d = 4$ cm]

SHORT ANSWER Type Questions

10. The mount everest is 8848 m above sea level. Estimate the acceleration due to gravity at this height, given that mean g on the surface of the earth is $9.8\ \text{m/s}^2$. [**Ans.** $9.77\ \text{m/s}^2$]

11. What will be the value of g at the bottom of sea 7 km deep? Diameter of the earth is 12800 km and g on the surface of the earth is $9.8\ \text{m/s}^2$. [**Ans.** $9.789\ \text{m/s}^2$]

12. If the earth was a perfect sphere of radius 6.37×10^{6} m, rotating about its axis with a period of 1 day, then how much would the acceleration due to gravity (g) differ from the poles to the equator? [**Ans.** $3.37\times10^{-2}\ \text{m/s}^2$]

LONG ANSWER Type I Questions

|3 Marks|

13. Calculate the earth's surface potential from the following data.

(i) Radius of the earth, $R = 6.63\times10^{6}$ m

(ii) Mean density of the earth, $\rho = 5.57\times10^{3}\ \text{kgm}^{-3}$

(iii) $G = 6.67\times10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$

[**Ans.** $-6.84\times10^{7}\ \text{J kg}^{-1}$]

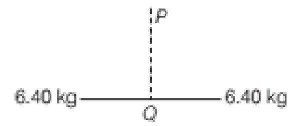
14. At a point above the earth, the gravitational potential is $-5.12\times10^{7}\ \text{J kg}^{-1}$ and acceleration due to gravity is $6.4\ \text{ms}^{-2}$. Assuming the mean radius of the earth to be 6400 km, calculate the height of this point above the surface of the earth. [**Ans.** 7600 km]

LONG ANSWER Type II Questions

15. Two equal masses of 6.40 kg are separated by a distance of 0.16 m. A small body is released from a point P equidistant from the two masses and at a distance of 0.06 m from the line joining them.

(i) Calculate the velocity of this body when it passes through Q.

(ii) Calculate the acceleration of this body at P and Q if its mass is 0.1 kg.



[**Ans.** (i) $6.53\times10^{-5}\ \text{ms}^{-1}$ (ii) $5.12\times10^{-8}\ \text{ms}^{-2}$]

16. What do you understand by gravitational field, intensity of gravitational field? Prove that gravitational intensity at a point is equal to the acceleration due to gravity at that point.

17. A body weighs 90 kg on the surface of the earth. How much will it weigh on the surface of the mars whose mass is $\frac{1}{9}$ th and radius $\frac{1}{2}$ of that of the earth? [**Ans.** 40 kg]

18. The Earth-moon distance is 3.8×10^{5} km. The mass of the earth is 81 times that of moon. Determine the distance from the earth to the point where the gravitational fields due to the earth and the moon cancel out. [**Ans.** 3.42×10^{5} km]

19. What should be the angular speed of earth, so that bodies lying on equator may appear weight less? [**Ans.** $1.25\times10^{-3}\ \text{rad s}^{-1}$]

20. Find the intensity of gravitational field at a point lying at a distance x from the centre on the axis of a ring of radius 'a' and mass 'M'.

|TOPIC 3|

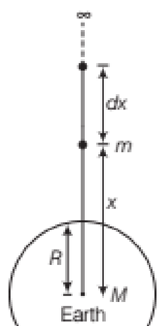
Escape Speed, Geostationary and Solar Satellites

ESCAPE SPEED

Escape speed on the earth (or any other planet) is defined as the minimum speed with which a body should be projected vertically upwards from the surface of the earth so that it just escapes out from gravitational field of the earth and never return on its own.

e.g. If you fire a projectile upward, usually it will slow down, stop momentarily and return to the earth. A certain minimum initial speed will cause it to move upward forever and coming to rest only at infinity.

Consider a body of mass m lying at a distance x from the centre of the earth. Let M be the mass and R be radius of the earth as shown in figure. According to Newton's law of gravitation, **the gravitational force F of attraction between the body and the earth** is given by



Calculation for escape speed

$$F = G\frac{Mm}{x^2}$$

If dW be the work done to displace the body through a small distance dx, then

$$dW = Fdx = \frac{GMm}{x^2}dx$$

Total work done in taking the body from the surface of the earth i.e. $x = R$ to $x = \infty$ is given by

$$W = \int dW = \int_R^\infty \frac{GMm}{x^2}dx$$

or $$W = GMm\int_R^\infty \frac{1}{x^2}dx$$

[$\because$ GMm is the constant quantity]

$\therefore$ $$W = GMm\int_R^\infty x^{-2}dx = GMm\left(\frac{x^{-2+1}}{-2+1}\right)_R^\infty$$

or $$W = GMm\left[\frac{x^{-1}}{-1}\right]_R^\infty = -GMm\left[\frac{1}{x}\right]_R^\infty$$

$$= -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$W = \frac{GMm}{R} \qquad \left[\because \frac{1}{\infty} = 0\right]$$

If v_e is the required escape speed of the body, then kinetic energy imparted to the body $= \frac{1}{2}mv_e^2$

$\therefore$ $$\frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

or $$v_e^2 = \frac{2GM}{R}$$

$$v_e = \sqrt{\frac{2GM}{R}} \qquad \ldots\text{(i)}$$

Thus, the escape speed of a body depends upon the mass and radius of the planet from which the body is projected.

$$g = \frac{GM}{R^2}$$

$\Rightarrow$ $$GM = gR^2$$

$\therefore$ $$v_e = \sqrt{\frac{2gR^2}{R}}$$

or $$v_e = \sqrt{2gR} \qquad \ldots\text{(ii)}$$

or $$\boxed{\text{Escape velocity, } v_e = \sqrt{2gR}}$$

where, R is the radius of the earth.

From Eq. (i),

$$v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \times \rho}$$

where, ρ is the mean density of the earth.

$$\boxed{\text{Escape velocity, } v_e = R\sqrt{\frac{8}{3}\pi G\rho}} \qquad \ldots\text{(iii)}$$

Escape Speed from Principle of Conservation of Energy

Let the object reach at infinity and its speed be v_f. The energy of an object is the sum of potential energy and kinetic energy. Then, the total energy of the projectile at infinity is given by

$$E(\infty) = W_1 + \frac{mv_f^2}{2} \qquad \ldots\text{(i)}$$

where, W_1 is gravitational potential energy of the object at infinity.

If the object was thrown initially with a speed v_i from a point at a distance $(h+R)$ from the centre of the earth, then its initial energy is given by

$$E(h+R)=\frac{1}{2}mv_i^2-\frac{GmM}{h+R}+W_1 \qquad \ldots\text{(ii)}$$

According to principle of energy conservation Eqs.(i) and (ii) are equal. So, $\frac{mv_i^2}{2}-\frac{GmM}{(h+R)}=\frac{mv_f^2}{2}$

If $\frac{mv_f^2}{2}$ is zero, then $\frac{mv_i^2}{2}-\frac{GmM}{h+R}=0$

Thus, the minimum speed required for an object to reach infinity i.e. escape from the earth corresponds to

$$\frac{1}{2}m(v_i^2)_{\min}=\frac{GmM}{h+R}$$

If the object is thrown from the surface of the earth, h is zero, so we get

$$(v_i)_{\min}=\sqrt{\frac{2GM}{R}}$$

Using the relation, $g=GM/R^2$, we get

$$\boxed{\text{Minimum speed, } (v_i)_{\min}=\sqrt{2gR}}$$

If $g=9.8\ \text{m/s}^2$ and $R=6.4\times10^6\text{m}$, then

$$(v_i)_{\min}=11.2\ \text{km/s}$$

EXAMPLE |1| Escape Velocity on Solar System

Calculate the escape speed of a body from the solar system from following data

(i) Mass of the sun $=2\times10^{30}$ kg.

(ii) Separation of the earth from the sun $=1.5\times10^{11}$ m.

Sol. If M be the mass of the sun and R be the distance of the earth from the sun, then escape velocity,

$$v_e=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2\times6.67\times10^{-11}\times2\times10^{30}}{1.5\times10^{11}}}\ \text{m s}^{-1}$$

$$=\sqrt{\frac{4\times6.67}{1.5}}\times10^4\ \text{ms}^{-1}$$

$$=4.217\times10^4\ \text{ms}^{-1}$$

$$v_e=42.17\ \text{kms}^{-1} \qquad [1\ \text{km}=1000\ \text{m}]$$

$\therefore$ The escape speed for solar system is 42.17 kms^{-1}.

Note

Orbital velocity (v_0) of a satellite is the velocity required to put the satellite into its orbit around the earth, $v_0=\sqrt{gR}$, i.e. $v_e=\sqrt{2}v_0$

EXAMPLE |2| A Rockey Sphere

Estimate the size of a rockey sphere with a density of $3.0\ \text{g/cm}^3$ from the surface of which you could barely throw a golf ball and have it never back. (assume your best throw is 40 m/s).

Sol. Let us consider that the rockey sphere has mass M and radius R. The escape speed for such a sphere is given by

$$v_e=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2G\left(\frac{4\pi}{3}\right)R^3\rho}{R}}=\sqrt{\frac{8\pi G\rho}{3}}R$$

or $$R=v_e\sqrt{\frac{3}{8\pi G\rho}}$$

Here, $\rho=3.0\ \text{g/cm}^3=3.0\times10^3\ \text{kg/m}^3$, $v_e=40$ m/s

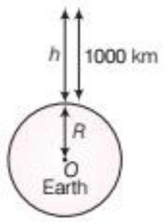
Thus, $$R=40\sqrt{\frac{3}{8\times3.14\times6.67\times10^{-11}\times3\times10^3}}\ \text{m}$$

or $$R=40\times772.6\ \text{m}=30904\ \text{m}=30.904\ \text{km}$$

EXAMPLE |3| Escape velocity for an atmospheric particle

Calculate the escape velocity for an atmospheric particle 1000 km above the earth's surface, given that the radius of the earth $=6.4\times10^6$ m and acceleration due to gravity on the surface of the earth $=9.8\ \text{m s}^{-2}$.

Sol.



Here, Height, $h=1000$ km, $R=6.4\times10^6$ m, $g=9.8\ \text{ms}^{-2}$

$$h=1000\times1000=10^6\ \text{m}$$

Escape velocity at a height h above the earth's surface

$$v_e=\sqrt{2g_h(R+h)}$$

Substitute the value of g_h in the above formula, we get

$$g_h=\frac{gR^2}{(R+h)^2}$$

$$\Rightarrow\ v_e=\sqrt{\frac{2\times gR^2}{(R+h)^2}(R+h)}=\sqrt{\frac{2gR^2}{R+h}}$$

$$v_e=\sqrt{\frac{2\times9.8\times(6.4\times10^6)^2}{(6.4+1)\,10^6}}\ \text{ms}^{-1}$$

$$=10.42\ \text{kms}^{-1}$$

EARTH SATELLITES

A satellite is a body which is constantly revolving in an orbit around a comparatively much larger body.

e.g. The moon is a natural satellite while INSAT-1B is an artificial satellite of the earth. Condition for establishment of satellite is that the centre of orbit of satellite must coincide with centre of the earth or satellite must move around great circle of the earth.

As we have already discussed that the centripetal force required for circular orbit is given by

$$F_c = \frac{mv^2}{R+h} \qquad \ldots(\text{i})$$

and is directeds toward the centre. This centripetal force is provided by the gravitational force which is expressed as,

$$F_g = \frac{GmM}{(R+h)^2} \qquad \ldots(\text{ii})$$

Equating both these Eqs. (i) and (ii), we get

$$v^2 = \frac{GM}{R+h}$$

Thus, v decreases as h increases. When, $h = 0$

$$\therefore \qquad v^2 = \frac{GM}{R} = gR \qquad \left[\because g = \frac{GM}{R^2}\right]$$

In every orbit, the satellite traverses a distance $2\pi(R+h)$ with speed v.

Then, its time period is given by

$$T = \frac{2\pi(R+h)}{v}$$

$$= \frac{2\pi(R+h)^{3/2}}{\sqrt{GM}} \qquad \ldots(\text{iii}) \left[\because v^2 = \frac{GM}{R+h}\right]$$

On squaring both sides in Eq. (iii), we get

$$\boxed{T^2 = k(R+h)^3} \qquad \left[\because k = \frac{4\pi^2}{GM}\right]$$

Thus, it is Kepler's law of periods. For a satellite, very close to surface of earth, h can be neglected in comparison to R. If $h = 0$ and T is T_0, then

$$T_0 = 2\pi\sqrt{R/g}$$

For earth, $R = 6400$ km and $g = 9.8\ \text{m/s}^2$

$$T_0 = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}} \cong 85\ \text{min}$$

EXAMPLE |4| Phobos and Delmos

The planet mars has two moons, phobos and delmos.

(i) Phobos has a period 7h, 39 min and an orbital radius of 9.4×10^3 km. Calculate the mass of mars.

(ii) Assume that earth and mars move in circular orbits around the sun with the martian orbit being 1.52 times the orbital radius of the earth. What is the length of the martian year in days?

Sol. (i) Given, $T = 7\ \text{h}\ 39\ \text{min} = 459\times60\ \text{s}$

$$R = 9.4\times10^3\ \text{km} = 9.4\times10^6\ \text{m}, \quad M_m = ?$$

$$\therefore \text{ Mass of mars, } M_m = \frac{4\pi^2}{G}\cdot\frac{R^3}{T^2}$$

$$= \frac{4\times(3.14)^2\times(9.4\times10^6)^3}{6.67\times10^{-11}\times(459\times60)^2}$$

$$= 6.48\times10^{23}\ \text{kg}$$

(ii) From Kepler's third law, $\dfrac{T_m^2}{T^2} = \dfrac{R_{MS}^3}{R_{ES}^3}$

where, R_{MS} is the mass-sun distance and R_{ES} is the earth-sun distance

$$\therefore \qquad T_m = (1.52)^{3/2}\times365 = 684\ \text{days}$$

EXAMPLE |5| Revolution of a Moon

Express the constant k in equation $T^2 = k(R+h)^3$ in days and kilometres. Given, $k = 10^{-13}\ \text{s}^2/\text{m}^3$. The moon is at a distance of 3.84×10^5 km from the orbit. Obtain its time period of revolution in days. **[NCERT]**

Sol. $k = 10^{-13}\ \text{s}^2/\text{m}^3$

$$= 10^{-13}\left[\frac{1}{(24\times60\times60)^2}\text{d}^2\right]\left[\frac{1}{(1/1000)^3\ \text{km}^3}\right]$$

$$= 1.33\times10^{-14}\ \text{d}^2/\text{km}^3$$

$$T^2 = k(R+h)^3 = 1.33\times10^{-14}\times(3.84\times10^5)^3 = 27.3\ \text{days}$$

ENERGY OF AN ORBITING SATELLITE

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of the earth) and kinetic energy (due to orbital motion).

If m is the mass of the satellite and v is its orbital velocity. Then, KE of the satellite

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{r}$$

$$\boxed{\text{Kinetic energy of satellite, } K = \frac{GMm}{2(R+h)}} \qquad \ldots(\text{i})$$

$$[\because r = R+h]$$

PE of the satellite, $U = mV$

$$\boxed{\text{Potential energy of satellite, } U = -\frac{GMm}{R+h}} \quad \ldots\text{(ii)}$$

Total mechanical energy of satellite, $E = K + U$

or $$E = \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$E = -\frac{GMm}{2(R+h)} \quad \ldots\text{(iii)}$$

$$\therefore \quad \boxed{\text{Total energy of satellite, } E = -\frac{GMm}{2(R+h)}}$$

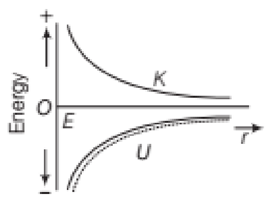
Satellites are always at finite distance from the earth and hence, their energy cannot be positive or zero.

Now, plot the graph between the kinetic, potential and total energies by considering the following conditions.

(i) Kinetic energy $(K) = -$ (total energy)

(ii) Potential energy $(U) = 2$ (total energy)

(iii) Potential energy $(U) = -2$ (kinetic energy) as shown in Fig (a).



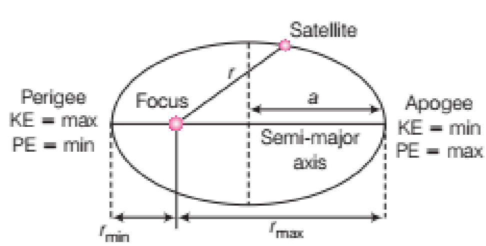
(a) Energy graph for a satellite

If the orbit of a satellite is elliptic as shown in Fig. (b),

then, total energy $(E) = \frac{-GMm}{2a}$

$= \text{constant},$

where a is semi-major axis.



(b) Energy distribution in elliptical orbit

EXAMPLE |6| Transferring the Satellite

A 400 kg satellite is in a circular orbit of radius 2R about the earth. How much energy is required to transfer it to a circular orbit of radius 4R ? What are the changes in the kinetic and potential energies? **[NCERT]**

Sol. Given

Mass of satellite, $m = 400$ kg

The initial energy is given by, $E_i = \frac{-GMm}{4R}$

Final energy is given by, $E_f = \frac{-GMm}{8R}$

$\therefore$ Change in the total energy is given by

$$\Delta E = E_f - E_i = \frac{-GMm}{8R} - \left(\frac{-GMm}{4R}\right)$$

$$\Delta E = \frac{GMm}{8R} = \left(\frac{GM}{R^2}\right)\frac{mR}{8}$$

$$= \frac{gmR}{8} = \frac{9.81 \times 400 \times 6.37 \times 10^6}{8}$$

$$\Delta E = 3.13 \times 10^9 \text{ J}$$

The kinetic energy is reduced,

$$\therefore \quad \Delta K = K_f - K_i = -3.13 \times 10^3 \text{ J}$$

Change in potential energy,

$$\Delta U = -2 \times \Delta E = -2 \times 3.13 \times 10^9 = -6.25 \times 10^9 \text{ J}$$

WEIGHTLESSNESS

The weight of a body is the force with which it is attracted towards the centre of the earth. When a body is stationary with respect to the earth, its weight equals to the gravity. This weight of the body is known as its **static** or **true weight**.

At one particular position, the two gravitational pulls may be equal and opposite and the net pull on the body becomes zero. This is zero gravity region or the null point and the body is said to be **weightless**.

The state of weightlessness can be observed in the following situations

(i) When objects fall freely under gravity.

e.g. A lift falling freely

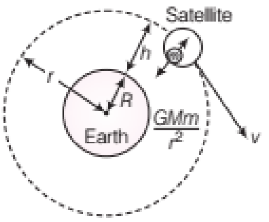
(ii) When a satellite revolves in its orbit around the earth.

(iii) When bodies are at null points in outer space.

Weightlessness in a Satellite

A satellite which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The acceleration of satellite is GM/r^2 towards the centre of the earth as shown in figure.

If a body of mass m is placed on a surface inside a satellite moving around the earth. Then, forces on the body are



Weightlessness in a satellite

The gravitational pull of the earth $= \dfrac{GMm}{r^2}$

The reaction by the surface $= N$

By Newton's law, $\dfrac{GMm}{r^2} - N = ma$

$$\frac{GMm}{r^2} - N = m\left(\frac{GM}{r^2}\right)$$

$$N = 0$$

Thus, the surface does not exert any force on the body and hence, its apparent weight is zero.

Note

Condition of weightlessness can be experienced only when the mass of satellite is negligible so that it does not produce its own gravity.

e.g. Moon is a satellite of the earth but due to its own weight, it applies gravitational force of attraction on the body placed on its surface and hence, weight of the body will not be equal to zero at the surface of the moon.

The experiment of simple pendulum cannot be performed in the weightlessness state because

$$T = 2\pi\sqrt{\frac{L}{g}} = \infty \text{ as } g = 0$$

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. An object is thrown from the surface of the moon. The escape speed for the object is

(a) $\sqrt{2g' R_m}$, where g' = acceleration due to gravity on the moon and R_m = radius of the moon

(b) $\sqrt{2g' R_e}$, where g' = acceleration due to gravity on the moon and R_e = radius of the earth

(c) $\sqrt{2gR_m}$, where g = acceleration due to gravity on the earth and R_m = radius of the moon

(d) None of the above

Sol. (a) Escape speed from the moon $= \sqrt{2g'R_m}$

where,

g' = acceleration due to gravity on the surface of moon.

R_m = radius of the moon

2. The escape velocity of a body from the earth is v_e. If the radius of earth contracts to 1/4 th of its value, keeping the mass of the earth constant, escape velocity will be

(a) doubled (b) halved

(c) tripled (d) unaltered

Sol. (a) Given, escape speed $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$

where, M_e = mass of the earth, R_e = radius of the earth

Now, radius of earth $= R' = R_e/4$

$$\Rightarrow v'_e = \sqrt{\frac{2GM}{R'}} = \sqrt{4\left(\frac{2\,GM}{R_e}\right)} = 2\sqrt{\frac{2GM}{R_e}}$$

or $v'_e = 2\,v_e$

3. The kinetic energy of the satellite in a circular orbit with speed v is given as

(a) $\text{KE} = \dfrac{-GmM_e}{2(R_e + h)}$ (b) $\text{KE} = \dfrac{GmM_e}{(R_e + h)}$

(c) $\text{KE} = \dfrac{GmM_e}{2(R_e + h)}$ (d) $\text{KE} = -\dfrac{1}{2}mv^2$

Sol. (c) KE of satellite $= \dfrac{1}{2}mv^2$

$$= \frac{1}{2}m\left(\sqrt{\frac{GM_e}{(R_e + h)}}\right)^2$$

$$= \frac{1}{2}\frac{GmM_e}{(R_e + h)}$$

4. Satellites orbitting the earth have finite life and sometimes debris of satellites fall to the earth. This is because [NCERT Exemplar]

(a) the solar cells and batteries in satellites run out

(b) the laws of gravitation predict a trajectory spiralling inwards

(c) of viscous forces causing the speed of satellite and hence height to gradually decrease

(d) of collisions with other satellites

Sol. (c) As the total energy of the earth satellite bounded system is negative $\left(\dfrac{-GM}{2a}\right)$.

where, a is radius of the satellite and M is mass of the earth.
Due to the viscous force acting on satellite, energy decreases continuously and radius of the orbit or height decreases gradually.

5. The orbital velocity of a satellite orbiting near the surface of the earth is given by

(a) $v = \sqrt{gR_e}$, where $g = \dfrac{GM_e}{R_e^2}$

(b) $v = \sqrt{gR_e}$, where $g = \dfrac{GM_e}{R_e}$

(c) $v = \sqrt{\dfrac{gh}{R_e}}$, where $g = \dfrac{GM_e}{R_e^2}$

(d) $v = \sqrt{gh}$, where $g = \dfrac{GM_e}{R_e}$

Sol. (a) Orbital velocity of satellite, $v = \sqrt{\dfrac{GM_e}{(R_e + h)}}$

If the satellite is close to the surface of the earth, $h = 0$

$$\Rightarrow \quad v = \sqrt{\frac{GM_e}{R_e}} \Rightarrow v = \sqrt{\left(\frac{GM_e}{R_e^2}\right) R_e}$$

$$= \sqrt{gR_e}, \qquad \left[\because g = \frac{GM_e}{R_e^2}\right]$$

VERY SHORT ANSWER Type Questions

6. Can we determine the mass of a satellite by measuring its time period?

Sol. No, we cannot determine the mass of a satellite by measuring its time period.

7. Why is the atmosphere much rarer on the moon than on the earth?

Sol. The value of escape velocity on the moon is small as compared to the value of the earth only 2.5 kms^{-1}. So, the molecules of air escape easily from the surface of moon, hence there is no atmosphere.

8. What velocity will you give to a donkey and what velocity to a monkey so that both escape the gravitational field of the earth?

Sol. The escape velocity does not depend upon the mass of the body.

$$v_e = \sqrt{2gR}$$

9. Two satellites are at different heights. Which would have greater velocity?

Sol. $v_o \propto \dfrac{1}{\sqrt{r}}$; so the satellite at smaller height would possess greater velocity.

10. A thief with a box in his hand jumps from the top of a building. What will be the load experienced by him during the state of free fall?

Sol. During the state of free fall, his acceleration is equal to the acceleration due to gravity. So, the thief will be in state of weightlessness. Hence, the load experienced by him will be zero.

11. Is it possible to put an artificial satellite in an orbit such a way that it always remains visible directly over New Delhi?

Sol. It is not possible. This is because New Delhi is not in the equatorial plane.

12. Does the speed of a satellite remain constant in a particular orbit (circular)?

Sol. Yes, as $v = \sqrt{\dfrac{GM}{r}}$, v depends only upon r. For a particular orbit, r is constant and so is v.

13. How much energy is required by a satellite to keep it orbiting? Neglect air resistance.

Sol. No energy is required. This is because the work done by centripetal force is zero.

14. An artificial satellite revolving around the earth does not need any fuel. On the other hand, the aeroplane requires fuel to fly at a certain height. Why?

Sol. The satellite move in air-free region, while the aeroplane has to overcome air resistance.

15. Assume that an artificial satellite release a bomb. Would the bomb ever strike the earth if the effect of air resistance is neglected?

Sol. The bomb would merely act as another satellite. It would never hit the earth due to state of weight lessness.

SHORT ANSWER Type Questions

16. Show that the orbital velocity of a satellite revolving the earth is 7.92 km s^{-1}.

Sol. Orbital velocity, $v_o = \sqrt{gR}$

$$= \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7.92 \text{ kms}^{-1}$$

17. The orbiting velocity of an earth-satellite is 8 kms^{-1}. What will be the escape velocity?

Sol. Escape velocity, $v_e = \sqrt{2}\, v_o$

$$v_e = \sqrt{2} \times 8$$

$$= 11.31 \text{ kms}^{-1}$$

18. **A satellite does not need any fuel to circle around the earth. Why?**

Sol. The gravitational force between satellite and the earth provides the centripetal force required by the satellite to move in a circular orbit. The satellite orbits around earth at such a higher height where air friction is neglible.

19. **If the kinetic energy of a satellite revolving around the earth in any orbit is doubled, then what will happen to it?**

Sol. The total energy of a satellite in any orbit, $E = -K$, where K is its KE in that orbit.

If its kinetic energy is doubled, i.e. an additional kinetic energy (K) is given to it, $E = -K + K = 0$ and the satellite will leave its orbit and go to infinity.

20. **On what factor does the escape speed from a surface depend?**

Sol. Value of escape speed at the surface of a planet is given by the relation

$$v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{2\,gR}$$

Thus, the value of escape speed from the surface of a planet depends upon (*i*) value of acceleration due to gravity g at the surface and (*ii*) the size (i.e. radius) R of the planet only. It is independent of all other factors.

e.g. The mass and size of the body to be projected, angle of projection, etc.

21. **An astronaut, by mistake, drops his food packet from an artificial satellite orbiting around the earth. Will it reach the surface of the earth? Why?**

Sol. The food packet will not fall on the earth. As the satellite as well as astronaut were in a state of weightlessness, hence, the food packet, when dropped by mistake, will also start moving with the same velocity as that of satellite and will continue to move along with the satellite in the same orbit.

22. **If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, then what will happen to the satellite?**

Sol. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, satellite will not be able to revolve around the earth. Instead, the satellite will start moving along a straight line tangentially at that point on its orbit, where it is at the time of gravitational force becoming zero.

23. **Which of the following symptoms is likely to affect an astronaut in space (a) swollen feet (b) swollen face, (c) headache, (d) orientational problem.** **[NCERT]**

Sol. (a) We know that the legs carry the weight of the body in the normal position due to gravity pull. The astronaut in space is in weightlessness state. Hence, swollen feet may not affect his working.

(b) In the conditions of weightlessness, the face of the astronaut is expected to get more supply. Due to it, the astronaut may develop swollen face. As eyes, ears, nose, mouth etc., are all embedded in the face, hence swollen face may affect to great extent the seeing/hearing/smelling/eating capabilities of the astronaut in space.

(c) Headache is due to mental strain. It will persist whether a person is an astronaut in space or he is on earth. It means headache will have the same effect on the astronaut in space as on a person on earth.

(d) Space also has orientation. We also have the frames of reference in space. Hence, orientational problem will affect the astronaut in space.

24. **The escape speed on the earth is 11.2 km/s. What is its value for a planet having double the radius and eight times the mass of the earth?**

Sol. v_P (escape speed on a planet) $= \sqrt{\frac{GM_P}{R_P}}$

v_e (escape speed on the earth) $= \sqrt{\frac{GM_e}{R_e}}$

Clearly, $$\frac{v_P}{v_e} = \sqrt{\frac{M_P}{M_e} \times \frac{R_e}{R_P}} = \sqrt{8 \times \frac{1}{2}} = 2$$

or $$v_P = 2v_e = 22.4 \text{ km/s}$$

25. **The earth is acted upon by the gravitational attraction of the sun. Why don't the earth fall into the sun?**

Sol. The earth is orbiting round the sun in a stable orbit (nearly circular) such that the gravitational attraction of the sun just provides the required centripetal force to the earth for its orbital motion. So, net force on the earth is zero and consequently, the earth does not fall into the sun.

26. **The asteroid pallas has an orbital period of 4.62 yr. Find the semi-major axis of its orbit. Given, $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}, mass of the sun $= 1.99 \times 10^{30}$kg and 1 yr $= 3.156 \times 10^7$ s.**

Sol. $$T^2 = \frac{4\pi^2 a^3}{GM_s}$$

or $$a = \left[\frac{GM_s T^2}{4\pi^2}\right]^{1/3}$$

Substituting values, we get

$$a = 4.15 \times 10^{11} \text{ m}$$

27. Does the change in gravitational potential energy of a body between two given points depend upon the nature of path followed, why?

Sol. The change in gravitational potential energy of a body between two given points depends only upon the position of the given points and is independent of the path followed. is due to the fact that the gravitational force is a conservative force and work done by a conservative force depends only on the position of initial and final points and is independent of path followed.

LONG ANSWER Type I Questions

28. Define period of revolution. Derive an expression of period of revolution or time period of satellite.

Sol. Period of revolution of a satellite is the time taken by the satellite to complete one revolution round the earth. It is denoted by T.

$$\therefore \quad T = \frac{\text{Circumference of circular orbit}}{\text{Orbital velocity}}$$

$$\text{or} \quad T = \frac{2\pi r}{v_o}$$

$$\text{or} \quad T = \frac{2\pi(R+h)}{v_o} \quad [\because r = R+h]$$

$$\text{or} \quad T = 2\pi(R+h)\sqrt{\frac{R+h}{GM}} \quad \left[\because v_o = \sqrt{\frac{GM}{R+h}}\right]$$

$$\text{or} \quad T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$\text{Also,} \quad T = 2\pi\sqrt{\frac{(R+h)^2(R+h)}{GM}}$$

$$\text{or} \quad T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$\because \quad gR^2 = GM$$

$$\therefore \quad T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

29. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expanded to rocket, the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg, mass of the earth, $M = 6.0 \times 10^{24}$ kg, radius of the earth $= 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. **[NCERT]**

Sol. Mass of the earth, $M = 6.0 \times 10^{24}$ kg

Mass of the satellite, $m = 200$ kg

Radius of the earth, $R = 6.4 \times 10^6$ m

Height of the satellite above the earth's surface,

$$h = 400 \text{ km} = 0.4 \times 10^6 \text{ m}$$

Radius of the orbit of the satellite, $r = R + h$

$$= 6.4 \times 10^6 + 0.4 \times 10^6 = 6.8 \times 10^6 \text{ m}$$

Total energy of the satellite,

$$E = -\frac{GMm}{2r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{2 \times 6.8 \times 10^6}$$

$$= -5.9 \times 10^9 \text{ J}$$

Negative total energy denoted that the satellite is round to the earth. Therefore, to pull the satellite out of the earth's gravitational influence, energy required $= 5.9 \times 10^6$ J. **[1/2]**

30. Viscous force increase the velocity of a satellite. Discuss.

Sol. Imagine a satellite of mass m moving with a velocity v in an orbit of radius r around a planet of mass M.

PE of the satellite, $U = -\dfrac{GMm}{r}$

KE of the satellite, $K = \dfrac{1}{2}mv^2 = \dfrac{GMm}{2r}$ $\quad [\text{as } v = \sqrt{GM/r}]$

Total energy of the satellite, i.e.

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

For the sake of clarity, take $\dfrac{GMm}{2r} = x$

Clearly, $\quad U = -2x,\ K = x,\ E = -x$

The orbiting satellite loses energy due to viscous force acting on it due to atmosphere and as such it loses height.

Let the new orbital radius be $\dfrac{r}{2}$ (say)

Clearly, $\quad U' = -4x,\ K' = 2x$

$$E' = -2x$$

Clearly, $E' < E$, $U' < U$ and $K' > K$. Since, kinetic energy has increased, the velocity of the satellite increases.

31. Does the escape speed of a body from the earth depend on

(i) mass of the body

(ii) the location from where it is projected

(iii) the direction of projection

(iv) the height of the location from where the body is launched? **[NCERT]**

Sol. (i) No, escape velocity is independent of the mass of the body.

(ii) Yes, escape velocity depends (through slightly) on the location from where the body is projected because with location g changes and so should $v_e \ (= \sqrt{2gR})$ change.

(iii) No, escape velocity is independent of the direction of projection.

(iv) Yes, escape velocity depends (through slightly) on the height of location from where the body is projected as g depends on height.

32. The escape speed of a projectile on the earth's surface is 11.2 km/s. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets. **[NCERT]**

Sol. Let v_e be the escape velocity and v be the velocity of the body outside the gravitational field of the earth.

According to law of conservation of energy,

Initial KE of the body = energy spent by the body in crossing the earth's gravitational field + kinetic energy left with the body once outside the earth's gravitational field, i.e.

$$\frac{1}{2}m(3v_e)^2 = \frac{1}{2}mv_e^2 + \frac{1}{2}mv^2$$

or $\frac{9}{2}mv_e^2 = \frac{1}{2}mv_e^2 + \frac{1}{2}mv^2$ or $v^2 = 8v_e^2$

or $v = \sqrt{8v_e^2} = 2\sqrt{2}v_e$

As $v_e = 11.2$ km/s $\Rightarrow v = 2\sqrt{2} \times 11.2$ km/s

or $v = 31.7$ km/s

33. How will you weigh the sun i.e. estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km. **[NCERT]**

Sol. We can weight the sun i.e. find its mass (M) using Kepler's third law of planetary motion according to which

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \text{ or } M = \frac{4\pi^2 r^3}{GT^2}$$

Here, r (radius of the earth's orbit around the sun)

$= 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

T (time period of earth's revolution around the sun)

$= 1\text{y} = 3.156 \times 10^7$ s

Thus, $M = \dfrac{39.5\,(1.5 \times 10^{11}\text{ m})^3}{(6.67 \times 10^{-11}\text{ Nm}^2/\text{kg}^2)(3.156 \times 10^7\text{ s})^2}$

[as $5\pi^2 = 39.5$]

$M = 2.0 \times 10^{30}$ kg

34. Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50000 ly from the galactic centre take to complete one revolution? **[NCERT]**

Sol. Mass of the galaxy, $m = (2.5 \times 10^{11})$ solar mass

$= (2.5 \times 10^{11})(2 \times 10^{30}\text{ kg})$

$= 5 \times 10^{41}$ kg

Radius of the path of the star, $r = 50000$ ly

$= 50000(9.46 \times 10^{15}\text{ m})$

$= 4.73 \times 10^{20}$ m

If T is the time taken by the star to complete one revolution, then $T = \left(\dfrac{4\pi^2 r^3}{Gm}\right)^{1/2}$

$$T = \left[\frac{39.5\,(4.73 \times 10^{20}\text{ m})^3}{(6.67 \times 10^{-11}\text{ Nm}^2/\text{kg}^2)(5 \times 10^{41}\text{ kg})}\right]^{1/2}$$

$T = 11.194 \times 10^{15}$ s

or $T = \dfrac{11.194 \times 10^{15}}{3.156 \times 10^7}\text{y} = 3.55 \times 10^8\text{y}$

[as $1\text{y} = (365.24 \times 24 \times 60 \times 60)\text{ s} = 3.156 \times 10^7$ s]

35. Choose the correct alternatives.

(i) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of the kinetic/potential energy.

(ii) The energy required to rocket an orbiting satellite out of the earth's gravitational influence is more/less than the energy required in project a stationary object at the same height (as the satellite) of the earth's influence. **[NCERT]**

Sol. (i) Potential energy of satellite, $U = -\dfrac{GMm}{r}$

Kinetic energy of satellite, $K = \dfrac{GMm}{2r}$

Total energy, $E = U + K$

$$= \frac{-GMm}{r} + \frac{GMm}{2r}$$

$$= \frac{-GMm}{2r} = -K$$

i.e. negative of kinetic energy.

(ii) The energy required to rocket an orbiting satellite out of gravitational influence is less than the energy required to project a stationary object, because in case of orbiting satellite, the gravitational pull of the earth acting on it is balanced by centripetal force, so work is required only in rocketing it (no work is required against the gravitational pull).

36. A comet orbits the sun in highly elliptical orbit. Does the comet has a constant

(i) linear speed,

(ii) angular speed,

(iii) angular momentum,

(iv) kinetic energy,

(v) potential energy and

(vi) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the sun. **[NCERT]**

Sol. (i) According to law of conservation of angular momentum, $L = mvr$ = constant, therefore the comet moves faster when it is close to the sun and moves slower when it is farther away from the sun. Therefore, the speed of the comet does not remain constant.

(ii) As the linear speed varies, the angular speed also varies. Therefore, angular speed of the comet does not remain constant.

(iii) As no external torque is acting on the comet, therefore, according to law of conservation of angular momentum, the angular momentum of the comet remain constant.

(iv) Kinetic energy of the comet $= \frac{1}{2}mv^2$

As the linear speed of the comet changes, its kinetic energy also changes. Therefore, its KE does not remain constant.

(v) Potential energy of the comet changes as its kinetic energy changes.

(vi) Only angular momentum and total energy of a comet remain constant throughout its orbit.

37. Calculate the change in the energy of a 500 kg satellite when it falls from an altitude of 200 km to 199 km. If this change takes place during one orbit. Calculate the retarding force on the satellite.
Given, mass of the earth $= 6 \times 10^{24}$ kg and radius of the earth = 6400 km

Sol. Given, $M_e = 6 \times 10^{24}$ kg, $r_e = 6400$ km

$$r_1 = 6400 + 200 = 6600 \text{ km} = 6.6 \times 10^6 \text{ m}$$
$$r_2 = 6400 + 199 = 6599 \text{ km} = 6.599 \times 10^6 \text{ m}$$

$$\text{Change in energy} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
$$= 6.67 \times 10^{-11} \times 6 \times 10^{24} \times 500 \left(\frac{1}{6.6 \times 10^6} - \frac{1}{6.599 \times 10^6}\right)$$
$$= 2 \times 10^{17} \,(1.5152 \times 10^{-7} - 1.5154 \times 10^7) \text{ J}$$
$$= -4 \times 10^6 \text{ J}$$

If this occurs during one orbit, then the energy lost = force × distance.

If we take the distance as being the circumference of one orbit. Then,

$$\text{Retarding force} = \frac{4 \times 10^6}{2\pi \times 6.6 \times 10^6}$$
$$= \frac{4 \times 10^6}{2 \times 6.6 \times 3.14 \times 10^6} = 0.1 \text{ N}$$

LONG ANSWER Type II Questions

38. A rocket is fired vertically with a speed of 5 km/s from the earth surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth $= 6.0 \times 10^{24}$ kg. Mean radius of the earth $= 6.4 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ Nm²/kg². **[NCERT]**

Sol. Here, speed of the rocket, $v = 5$ km/s $= 5 \times 10^3$ m/s

Mass of the earth, $M = 6.0 \times 10^{24}$ kg

Radius of the earth, $x = 6.4 \times 10^6$ m

Gravitational constant, $G = 6.67 \times 10^{-11}$ Nm²/kg²

Let m be the mass of the rocket and h be the maximum height gained by the rocket.

Change in potential energy of the rocket

= PE at a height h − PE at the surface of the earth

$$= -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$
$$= GMm\left[\frac{h}{R(R+h)}\right] \quad \text{...(i)}$$

Initial KE of rocket $= \frac{1}{2}mv^2$

Final KE of the rocket when at the height, $h = 0$

Thus, change in KE of the rocket $= \frac{1}{2}mv^2$ ...(ii)

According to law of conservation of energy, from Eqs. (i) and (ii), we get

$$GMm\left[\frac{h}{R(R+h)}\right] = \frac{1}{2}mv^2$$

or $$\frac{v^2}{2GM} = \frac{h}{R^2 + Rh}$$

or $$v^2R^2 + v^2Rh = 2GMh$$

or $$h(2GM - v^2R) = v^2R^2$$

or $$h = \frac{v^2R^2}{2GM - v^2R}$$

or $$h = \frac{(5 \times 10^3 \text{ m/s})^2 (6.4 \times 10^6 \text{ m})^2}{[\{2 \times (6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg})\} - \{(5 \times 10^3 \text{ m/s})^2 (6.4 \times 10^6 \text{ m})\}]}$$
$$= 1.6 \times 10^6 \text{ m}$$

Distance of the rocket from the centre of the earth

$$= 6.4 \times 10^6 \text{ m} + 1.6 \times 10^6 \text{ m} = 8 \times 10^6 \text{ m}$$

39. A rocket is fired vertically from the surface of the mars with the speed of 2 km/s. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of the mars before returning to it. Mass of the mars = 6.4×10^{23} kg; radius of the mars = 3395 km; $G = 6.67 \times 10^{-11}$ Nm^2/kg^2 **[NCERT]**

Sol. We are given that

Mass of the mars, $M = 6.4 \times 10^{23}$ kg

Radius of the mars, $R = 3395 \text{ km} = 3.395 \times 10^6$ m

Velocity of the rocket, $v = 2 \text{ km/s} = 2 \times 10^3$ m/s

Let h be the maximum height attained by the rocket.

Change in potential energy of rocket.

PE = final potential energy − initial potential energy

$$= -G\frac{Mm}{(R+h)} - \left(-G\frac{Mm}{R}\right) = -G\frac{Mm}{(R+h)} + G\frac{Mm}{R}$$

$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = GMm\frac{h}{R(R+h)}$$

Here, 20% of the kinetic energy of the rocket is lost due to Martian atmosphere.

KE of the rocket which is converted into its potential energy $= \frac{80}{100} \times \frac{1}{2}mv^2 = 0.4\, mv^2$

Applying law of conservation of energy,

$$\Rightarrow \quad GMm\frac{h}{R(R+h)} = 0.4\, mv^2$$

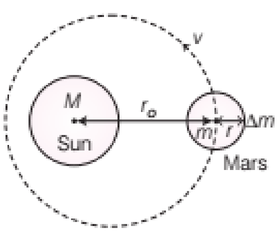
$$\Rightarrow \quad GM\frac{h}{R^2 + Rh} = 0.4\, v^2 \quad \text{or} \quad h = \frac{R^2}{\left(\frac{GM}{0.4\, v^2}\right) - R}$$

$$\Rightarrow \quad h = \frac{11.526 \times 10^{12}}{26.68 \times 10^6 - 3.395 \times 10^6} \text{ m}$$

$$\Rightarrow \quad h = 495 \times 10^3 \text{ m} = 495 \text{ km}$$

40. A spaceship is stationed on the mars. How much energy must be expanded on the spaceship to launch it out of the solar system. The mass of the spaceship = 1000 kg; mass of the sun = 2×10^{30} kg; mass of the mars = 6.4×10^{23} kg; radius of the mars = 3395 km; radius of the orbit of mars = 2.28×10^8 km, $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. **[NCERT]**

Sol. Given, mass of the sun, $M = 2 \times 10^{30}$ kg



Mass of the mars, $m = 6.4 \times 10^{23}$ kg

Mass of spaceship, $\Delta m = 1000$ kg

Radius of orbit of the mars, $r_o = 2.28 \times 10^{11}$ m

Radius of the mars, $r = 3.395 \times 10^6$ m

If v is the orbital velocity of mars.

$$\frac{mv^2}{r_o} = G\frac{Mm}{r_o^2} \quad \text{or} \quad v^2 = \frac{GM}{r_o}$$

Since, the velocity of the spaceship is the same as that of the mars.

Kinetic energy of the spaceship,

$$K = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}(\Delta m)\frac{GM}{r_o} = \frac{GM(\Delta m)}{2r_o}$$

$$\text{or } K = \frac{(6.67 \times 10^{-11})(2 \times 10^{30}) \times 1000}{2 \times (2.28 \times 10^{11})} \text{J} = 2.925 \times 10^{11} \text{ J}$$

Total potential energy of the spaceship.

U = potential energy of the spaceship due to its being in the gravitational field of the mars + potential energy of the spaceship due to its being in the gravitational field of the sun.

$$= -G\frac{m \times \Delta m}{r} - G\frac{M \times \Delta m}{r_o} = -G\Delta m\left(\frac{m}{r} + \frac{M}{r_o}\right)$$

$$\text{or } U = -6.67 \times 10^{-11} \times 10^3 \times \left(\frac{6.4 \times 10^{23}}{3.395 \times 10^6} + \frac{2 \times 10^{30}}{2.28 \times 10^{11}}\right) \text{J}$$

$$= -6.67 \times 10^{-8}\,(1.885 \times 10^{17} + 87.719 \times 10^{17}) \text{ J}$$

$$= -6.67\,(89.604 \times 10^9) \text{ J} = -5.977 \times 10^{11} \text{ J}$$

Total energy of the spaceship,

$$E = K + U = 2.925 \times 10^{11} \text{ J} - 5.997 \times 10^{11} \text{ J}$$

$$= -3.072 \times 10^{11} \text{ J} = -3.1 \times 10^{11} \text{ J}$$

Negative energy denotes that the spaceship is bound to the solar system.

Thus, the energy needed by the spaceship to escape from the solar system = 3.1×10^{11} J.

41. If one satellite of Jupiter has an orbital period of 1.769 days and the radius of the orbit is 4.22 $\times 10^8$m. Show that the mass of jupiter is about one-thousandth that of the sun. **[NCERT]**

Sol. Here, orbital period of the satellite of jupiter, $T_s = 1.769$ days $= 1.769 \times 24 \times 60 \times 60$ s

Orbital period of the earth, $T_e = 1\text{y} = 365.25$ d

Radius of the orbit of the satellite, $r_s = 4.22 \times 10^8$ m

Radius of the orbit of the earth, $r_e = 1 \text{ AU} = 1.496 \times 10^{11}$ m

According to Kepler's third law, $T^2 = \left(\frac{4\pi^2}{Gm}\right)r^3$

Let M_s and M_j represent the masses of the sun and the jupiter.

For the Sun-Earth system,

$$T_e^2 = \left(\frac{4\pi^2}{GM_s}\right)r_e^3$$

For Jupiter-Satellite system,

$$T_s^2 = \left(\frac{4\pi^2}{GM_j}\right) r_s^3$$

Thus, $$\frac{T_e^2}{T_s^2} = \left(\frac{M_j}{M_s}\right)\left(\frac{r_e}{r_s}\right)^3 \text{ or } \frac{M_j}{M_s} = \left(\frac{T_e}{T_s}\right)^2 \left(\frac{r_s}{r_e}\right)^3$$

or $$\frac{M_j}{M_s} = \left(\frac{365.25\text{d}}{1.769\text{ d}}\right)^2 \left(\frac{4.22\times 10^8\text{ m}}{1.496\times 10^{11}\text{ m}}\right)^3$$

$$= (4.26\times 10^4)(22.4\times 10^{-9})\times 10^{-3}$$

or $$M_j = 10^{-3} M_s$$

Thus, M_j (mass of the jupiter) $= \frac{1}{1000} M_s$ (mass of the sun)

42. A satellite is in an elliptic orbit around the earth with aphelion of $6R$ and perihelion of $2R$, where $R = 6400$ km is the radius of the earth. Find eccentricity of the orbit. Find the velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius $6R$? **[NCERT Exemplar]**

Sol. If r_a and r_p denote the distances of aphelion and perihelion of the elliptical orbit (of eccentricity) of the satellite, then from the geometry of the ellipse.

$$r_a = a(1+e) \text{ and } r_p = a(1-e)$$

[where, a is the semi-major axis of the ellipse]

As $r_a = 6R$ and $r_p = 2R$,

$$\frac{a(1+e)}{a(1-e)} = \frac{6R}{2R} = 3, \text{ where } e = \frac{1}{2}$$

If v_a and v_p are the velocities of the satellite (of mass m) at aphelion and perihelion respectively, then from the law of conservation of angular momentum,

$$mv_a r_a = mv_p r_p \text{ or } \frac{v_a}{v_p} = \frac{r_p}{r_a} = \frac{2R}{6R} = \frac{1}{3}$$

From the law of conservation of energy,

(KE + PE) at aphelion = (KE + PE) at perihelion

or $$\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = \frac{1}{2}mv_p^2 - \frac{GMm}{r_p}$$

or $$v_p^2 - v_a^2 = -2GM\left(\frac{1}{r_a} - \frac{1}{r_p}\right)$$

$$= 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

or $$v_p^2\left(1 - \frac{v_a^2}{v_p^2}\right) = 2GM\left(\frac{r_a - r_p}{r_p r_a}\right)$$

$$= 2GM\left(\frac{6R - 2R}{6R\times 2R}\right) = \frac{2}{3}\frac{GM}{R}$$

or $$v_p^2\left(1 - \frac{1}{9}\right) = \frac{2}{3}\frac{GM}{R} \text{ or } v_p = \sqrt{\frac{3}{4}\frac{GM}{R}}$$

$$= \frac{\sqrt{3}}{2}\sqrt{\frac{GM}{R}} = 0.8666\,(7.9\text{ km/s}) = 6.84\text{ km/s}$$

Also, $$v_a = \frac{v_p}{3} = \frac{6.84\text{ km/s}}{3} = 2.28\text{ km/s}$$

Velocity of the satellite in a circular orbit.

$$v_c = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{6R}} \quad [\text{as } r = 6R]$$

or $$v_c = \frac{1}{\sqrt{6}}\sqrt{\frac{GM}{R}} = 0.408\,(7.9\text{ km/s})$$

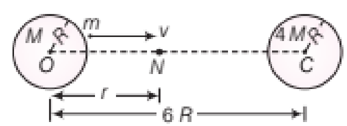
$$= 3.22\text{ km/s}$$

The amount by which velocity is to be increased in transferring the satellite into a circular orbit at apogee.

$$= v_c - v_a = 3.22\text{ km/s} - 2.28\text{ km/s} = 0.94\text{ km/s}$$

This is accomplished by suitably firing rockets from the satellites.

43. Two uniform solid spheres of equal radii R, but mass M and $4M$ have a centre of separation $6R$, as shown in figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere. **[NCERT]**



Sol. The two spheres exert gravitational forces on the projectile in mutually opposite directions. At the neutral point N, these two forces cancel each other. If $ON = r$, then

$$\frac{GMm}{r^2} = \frac{G(4M)\,m}{(6R - r)^2}$$

or $$(6R - r)^2 = 4r^2 \Rightarrow 6R - r = \pm 2r$$

or $$r = 2R \text{ or } -6R$$

The neutral point, $r = -6R$ is inadmissible.

$\therefore$ $$ON = r = 2R$$

It will be sufficient to project the particle m with a minimum speed v which enables it to reach the point N. Therefore, the particle m gets attracted by the gravitational pull of $4M$.

The total mechanical energy of m at surface of left sphere is

$$E_i = \text{KE of } m + \text{PE due to left sphere}$$
$$+ \text{PE due to right sphere}$$

$$= \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{4\,GMm}{5R}$$

At the neutral point, speed of the particle becomes zero. The energy is purely potential.

$\therefore$ E_N = PE due to left sphere + PE due to right sphere

$$= -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

By conservation of mechanical energy, $E_i = E_N$

or $$\frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{4\,GMm}{5R} = -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

or $$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3GM}{5R}$$

$\therefore$ $$v = \sqrt{\frac{3GM}{5R}}$$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. A particle is kept at rest at a distance R_e (earth's radius) above the earth's surface. The minimum speed with which it should be projected so that it does not return is (mass of earth = M_e)

(a) $\sqrt{\frac{6M_e}{4R_e}}$ (b) $\sqrt{\frac{GM_e}{2R_e}}$ (c) $\sqrt{\frac{GM_e}{R_e}}$ (d) $\sqrt{\frac{2\,GM_e}{R_e}}$

2. The ratio of the magnitude of potential energy and kinetic energy of a satellite is

(a) 1 : 2 (b) 2 : 1
(c) 3 : 1 (d) 1 : 3

3. Which one of the following statements is correct?

(a) The energy required to rocket an orbiting satellite out of earth's gravitational influence is more than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence
(b) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of potential energy
(c) The first artificial satellite sputnik I was launched in the year 2001
(d) The time period of rotation of the SYNCOMS (Synchronous communications satellite) is 24 hours.

4. The radius of the orbit of a satellite is r and its kinetic energy is K. If the radius of the orbit is doubled, then the new kinetic energy K' is

(a) $2K$ (b) $\frac{K}{2}$
(c) $4K$ (d) Data insufficient

Answer

1. (c)	2. (b)	3. (d)	4. (b)

VERY SHORT ANSWER Type Questions

5. Determine the speed of a body from the moon. Take the moon to be a uniform sphere of radius 1.74×10^6 m and mass 7.36×10^{22} kg. [**Ans.** 2.38 km/s]

6. Calculate the mass of the sun, given that distance between the sun and the earth is 1.49×10^{11} m and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. [**Ans.** 1.972×10^{30} kg]

7. Calculate the velocity with which a body projected from the surface of the moon may escapes from its gravitational pull. [**Ans.** 2.5 km/s]

8. Why do different planets have different escape speeds?

9. Can a pendulum vibrate in an artificial satellite?

SHORT ANSWER Type Questions

10. Calculate the ratio of the kinetic energy required to be given to a satellite so that it escape the gravitational field of earth to the kinetic energy required to put the satellite in a circular orbit just above the free surface of the earth. [**Ans.** 2]

11. Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. What is the speed of each particle? $\left[\textbf{Ans. } \frac{1}{2}\sqrt{\frac{Gm}{R}}\right]$

12. An artificial satellite moving in a circular orbit around the earth has a total energy E_0. What is its potential energy? [**Ans.** $U = 2E_0$]

13. Two satellites have their masses in the ratio of 3 : 1. The radii of their circular orbits are in the ratio of 1 : 4. What is the ratio of total mechanical energy of A and B? [**Ans.** 12:1]

LONG ANSWER Type I Questions

14. A 400 kg satellite is in circular orbit of radius $2R_E$ about the earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies?

[**Ans.** -3.13×10^6 J, -6.26×10^9 J]

15. The world's first artificial satellite (Sputnik-I) launched by USSR was circling the earth at a distance of 896 km. Calculate its orbital speed and period of revolution.

[**Ans.** 7.417 km/s, 1 h 43 min 3 s]

16. If the earth has a mass nine times and radius twice that of the planet mars, calculate the maximum velocity required by a rocket to pull out of the gravitational force of the mars.

[**Ans.** 5.28 km/s]

17. An artificial satellite is going round the earth, close to the surface. What is the time taken by it to complete one round? [**Ans.** 1.41 h]

LONG ANSWER Type II Questions

18. A spaceship is launched into a circular orbit close to the surface of the earth. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull. [**Ans.** 3.3 km/s]

19. If a satellite is revolving around a planet of mass M in elliptical orbit of major axis a, then show that the orbital speed of the satellite when it is at a distance r from the focus will be given by $v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$

20. The earth satellite makes a complete circular orbit in 1.5 h. Determine the altitude of satellite above the surface of the earth. [**Ans.** 277.2 km]

SUMMARY

- **Kepler's laws of planetary motion.**
  - (i) **Kepler's first law (law of orbit)** Every planet revolves around the sun in an elliptical orbit. The sun is situated at one focus of the ellipse.
  - (ii) **Kepler's second law (law of area)** The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time, i.e. the areal velocity of the planet around the sun is constant.
  - (iii) **Kepler's third law (law of period)** The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi-major axis of the elliptical orbit, i.e. $T^2 \propto r^3$, where r is the semi-major axis of the elliptical orbit of the planet around the sun.

    The period T and radius R of the circular orbit of a planet about the sun are related by
    $$T^2 = \left(\frac{4\pi^2}{GM_s}\right)R^3$$
    where, M_s is the mass of the sun. Most planets have nearly circular orbits about the sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis, a.
- **Universal law of gravitation** Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- **Universal law of gravitation**

  Gravitational force, $F = G\frac{m_1 m_2}{r^2}$

  Where, G = Constant of proportionality, = Universal Gravitational Constant

  $= 6.67 \times 10^{-11}$ Nm^{2}kg^{-2} (in SI unit)

  Dimensional formula of G = $[M^{-1}L^3T^{-2}]$
- **Principle of superposition** Resultant force, $F = -Gm_1\left[\frac{m_2}{r_{12}^2}\hat{r}_{21} + \frac{m_3}{r_{13}^2}\hat{r}_{31} + \ldots + \frac{m_n}{r_{1n}^2}\hat{r}_{n_1}\right]$

- The acceleration due to gravity (g) is related with gravitational constant (G) by the relation, $g=\frac{GM}{R^2}$ where, M and R are the mass and radius of the earth, respectively.
- **Variation of acceleration due to gravity**
  (i) **Effect of altitude,** $g'=\frac{gR^2}{(R+h)^2}$ and $g'=g\left(1-\frac{2h}{R}\right)$
  (ii) **Effect of depth,** $g'=g\left(1-\frac{d}{R}\right)$
- Intensity of Gravitational field at a point is, $I=\frac{F}{m}=\frac{GM}{r^2}$
- **Gravitational potential,** $v=-\frac{w}{m}=-\int I\cdot dr\ \left[\text{as } \frac{F}{m}=I\right]$
- **Gravitational potential energy,** U = gravitational potential $\times$ mass of body $=-\frac{GM}{r}\times m$.

$$v_e=\sqrt{\frac{2GM}{R}}=\sqrt{2gR}.$$ For Earth, the value of escape speed is 11.2 kms^{-1}.

For a point close to the Earth's surface, the escape speed and orbital speed are related as $v_e=\sqrt{2}\,v_o$, where v_o = the velocity required to put the satellite into its orbit.

(i) **Orbital speed of a satellite** is the speed required to put the satellite into given orbit around Earth. Orbital speed of satellite, when it is revolving around earth at height h is given by $v_o=R\sqrt{\frac{g}{R+h}}$

(ii) **Time period of satellite (T)** It is the time taken by satellite to complete one revolution around the earth.

$$T=\frac{2\pi(R+h)}{v_o}=\frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

(iv) **Total energy of satellite,**

$$E=-\frac{GMm}{(R+h)}+\frac{1}{2}mv_o^2=-\frac{GMm}{(R+h)}+\frac{1}{2}m\left(\frac{GM}{R+h}\right)=-\frac{GMm}{2(R+h)}$$

(v) **Binding energy of satellite** $=-E=\frac{GMm}{(R+h)}$.

- **Weightlessness** It is a situation in which the effective weight of the body becomes zero.

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. The law of areas can be interpreted as

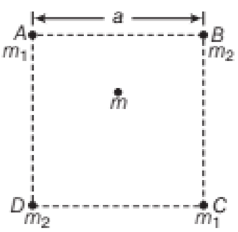
(a) $\frac{\Delta A}{\Delta t} = \text{constant}$ (b) $\frac{\Delta A}{\Delta t} = \frac{L}{m}$

(c) $\frac{\Delta A}{\Delta t} = \frac{1}{2}(r \times P)$ (d) $\frac{\Delta A}{\Delta t} = \frac{2L}{m}$

2. Three uniform spheres of mass M and radius R each are kept in such a way that each touches the other two. The magnitude of the gravitational force on any of the spheres due to the other two is

(a) $\frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$ (b) $\frac{3}{2}\frac{GM^2}{R^2}$ (c) $\frac{\sqrt{3}\,GM^2}{R^2}$ (d) $\frac{\sqrt{3}}{2}\frac{GM^2}{R^2}$

3. A point mass m is placed at the centre of the square $ABCD$ of side a units as shown below.



The resultant gravitational force on mass m due to masses m_1 and m_2 plant on the vertices of square is

(a) $\frac{Gm_1 m_2}{(a\sqrt{2})^2}$ (b) $\frac{2Gm(m_1 + m_2)}{a^2}$

(c) zero (d) $\frac{Gm(m_1 + m_2)}{(a\sqrt{2})^2}$

4. If the mass of the earth is doubled and its radius halved, then new acceleration due to the gravity g' is

(a) $g' = 4\,g$ (b) $g' = 8\,g$ (c) $g' = g$ (d) $g' = 16\,g$

5. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. Its radius in terms of radius of earth R will be

(a) $R/4$ (b) $R/2$

(c) $R/3$ (d) $R/8$

6. A particle of mass m is at the surface of the earth of radius R. It is lifted to a height h above the surface of the earth. The gain in gravitational potential energy of the particle is

(a) $\frac{mgh}{\left(1 - \frac{h}{R}\right)}$ (b) $\frac{mgh}{\left(1 + \frac{h}{R}\right)}$

(c) $\frac{mgh\,R}{(R+h)}$ (d) Both (b) and (c)

7. Three particles each of mass m are kept at vertices of an equilateral triangle of side L. The gravitational potential energy possessed by this system is

(a) $\frac{-Gm^2}{L}$ (b) $\frac{-3\,Gm^2}{L}$ (c) $-\frac{2\,Gm^2}{L}$ (d) $\frac{+3\,Gm^2}{L}$

8. If the gravitational potential energy at infinity is assumed to be zero, the potential energy at distance $(R_e + h)$ from the centre of the earth is

(a) $\text{PE} = \frac{GmM_e}{(R_e + h)}$ (b) $\text{PE} = \frac{-GmM_e}{(R_e + h)}$

(c) $\text{PE} = mgh$ (d) $\text{PE} = \frac{-GmM_e}{2(R_e + h)}$

9. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They **[NCERT Exemplar]**

(a) will not move around the sun, since they have very small masses compared to the sun

(b) will move in an irregular way because of their small masses and will drift away into outer space

(c) will move around the sun in closed orbits but not obey Kepler's laws

(d) will move in orbits like planets and obey Kepler's laws

10. Choose the wrong option. **[NCERT Exemplar]**

(a) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass

(b) That the gravitational mass and inertial mass are equal is an experimental result
(c) That the acceleration due to gravity on the earth is the same for all bodies is due to the equality of gravitational mass and inertial mass
(d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot

ASSERTION AND REASON

Direction (Q. Nos. 11-19) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

11. **Assertion** The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.

Reason Various region of the spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.

12. **Assertion** There is a popular statement regarding Cavendish: 'Cavendish weighed the earth'.

Reason The measurement of G by Cavendish's experiment, combined with the knowledge of g and R_E enables one to estimates M_E from equation.

$$g = \frac{GM_E}{R_E^2}$$

13. **Assertion** The velocity of the satellite decreases as its height above earth's surface increases and is maximum near the surface of the earth.

Reason The velocity of the satellite is inversely proportional to its height above earth's surface.

14. **Assertion** The total energy of the satellite is always negative irrespective of the nature of its orbit *i.e.*, elliptical or circular and it cannot be positive or zero.

Reason If the total energy is positive or zero, the satellite would leave its orbit.

15. **Assertion** As we go up the surface of the earth, we feel light weighed than on the surface of the earth.

Reason The acceleration due to gravity decreases on going up above the surface of the earth.

16. **Assertion** The escape speed for the moon is 2.3 kms^{-1} which is five times smaller than that for the earth.

Reason The escape speed depends on acceleration due to gravity on the moon and radius of the moon and both of them are smaller than that of earth.

17. **Assertion** Moon has no atmosphere.

Reason The escape speed for the moon is much smaller. Gas molecules, if formed on the surface of the moon having velocities larger than escape speed will escape the gravitational pull of the moon.

18. **Assertion** In the satellite everything inside is in a free fall.

Reason Free falling objects have no net upwards force acting on them.

19. **Assertion** An object is weightless when it is in free fall and this phenomenon is called phenomenon of weightlessness.

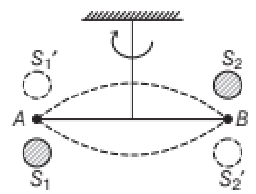
Reason In free fall, there is no upward force acting on the object.

CASE BASED QUESTIONS

Direction (Q. Nos. 20-21) *These questions are case study based questions. Attempt any 4 sub-parts from each question.*

20. **Cavendish's Experiment**

The figure shows the schematic drawing of Cavendish's experiment to determine the value of the gravitational constant. The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire.



Two large lead spheres are brought close to the small ones but on opposite sides as shown. The name of G from this experiment came to be $6.67 \times 10^{-11} N\text{-}m^2/kg^2$.

(i) The big spheres attract the nearby small ones by a force which is
(a) equal and opposite
(b) equal but in same direction
(c) unequal and opposite
(d) None of the above

(ii) The net force on the bar is

(a) non-zero
(b) zero
(c) Data insufficient
(d) None of the above

(iii) The net torque on the bar is

(a) zero
(b) non-zero
(c) F times the length of the bar, where F is the force of attraction between a big sphere and its neighbouring
(d) Both (b) and (c)

(iv) The torque produces twist in the suspended wire. The twisting stops when

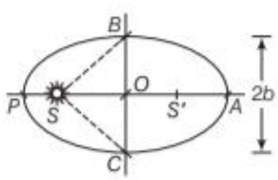
(a) restoring torque of the wire equals the gravitational torque
(b) restoring torque of the wire exceeds the gravitational torque
(c) the gravitational torque exceeds the restoring torque of the wire
(d) None of the above

(v) After Cavendish's experiment, there have been suggestions that the value of the gravitational constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,

(a) nothing will change
(b) we will become hotter after billions of years
(c) we will be going around but not strictly in closed orbits
(d) None of the above

21. Angular Momentum

Let the speed of the planet at the perihelion P in above figure be v_P and the Sun planet distance SP be r_P. The corresponding quantity at the aphelion A is (r_A, v_A). The mass of the planet in m_P.



(i) The magnitude of the angular momentum at P is

(a) $L_P = \dfrac{m_P r_P}{v_P}$ (b) $L_P = \dfrac{m_P v_P}{r_P}$

(c) $L_P = m_P r_P v_P$ (d) $L_P = m_P v_P$

(ii) The magnitude of the angular momentum at A is

(a) $L_A = m_P r_A v_A$ (b) $L_A = \dfrac{m_P r_A}{v_A}$

(c) $L_A = m_P v_A$ (d) $L_A = \dfrac{m_P v_A}{r_A}$

(iii) The vectors $\mathbf{r}_P$ and $\mathbf{v}_P$ are

(a) parallel (b) mutually perpendicular
(c) anti-parallel (d) coincident

(iv) The relation for conservation of angular momentum at P and A is

(a) $m_P r_P v_P = m_P r_A v_A$
(b) $\dfrac{m_P r_P}{v_P} = \dfrac{m_P r_A}{v_A}$
(c) $m_P v_P = m_P v_A$
(d) $\dfrac{m_P v_P}{r_P} = \dfrac{m_P v_A}{r_A}$

(v) The magnitude of the angular momentum as the planet goes round the sun is

(a) constant
(b) variable
(c) depends on magnitude of velocity
(d) increases as planet goes near to sun

Answer

1. (a)	2. (a)	3. (c)	4. (b)	5. (b)
6. (d)	7. (b)	8. (b)	9. (d)	10. (d)
11. (a)	12. (a)	13. (a)	14. (a)	15. (a)
16. (a)	17. (a)	18. (b)	19. (a)	
20. (i) (a)	(ii) (d)	(iii) (b)	(iv) (a)	(v) (c)
21. (i) (c)	(ii) (a)	(iii) (b)	(iv) (a)	(v) (a)

VERY SHORT ANSWER Type Questions

22 Calculate the force of attraction between two balls, each of mass 1 kg, when their centres are 10 cm apart. **[Ans.** 6.67×10^{-9} N]

23. If a planet exist whose mass and radius were both half those of the earth, what would be the value of acceleration due to gravity on its surface as compared to what it is on the surface of the earth? **[Ans.** 2]

24. What would happen to an artificial satellite, if its orbital velocity is slightly decreased due to some defects in it?

25. If the diameter of the earth becomes two times its present value and its mass remains unchanged, then how would the weight of an object on the surface of the earth be affected?
[**Ans.** Weight would become one-fourth]

SHORT ANSWER Type Questions

26. How far away from the surface of the earth does the acceleration due to gravity become 4% of its value on the surface of the earth?
[**Ans.** 25600 km]

27. Find the percentage decrease in the weight of a body when taken 10 km below the surface of the earth. [**Ans.** 0.25]

28. How much below the surface of the earth does acceleration due to gravity become 1% of the value at the earth's surface? [**Ans.** 6.32×10^3 km]

29. Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh 3/5 as much as present. [**Ans.** 5 kms^{-1}]

30. State and explain Kepler's laws of planetary motion. Also, name the physical quantities which remain constant during the planetary motion.

31. Prove that the moon would depart forever if, its speed were increased by nearly 42%.

LONG ANSWER Type I Questions

32. A body is projected vertically upwards from the bottom of a crater of the moon of depth $\frac{R}{100}$, where R is the radius of the moon, with a velocity equal to the escape velocity on the surface of the moon. Calculate the maximum height attained by the body from the surface of the moon. [**Ans.** 100 R]

33. A body is projected vertically upwards from the surface of the earth so as to reach a height equal to the radius of the earth. Neglecting resistance due to air, calculate the initial speed which should be imparted to the body.
[**Ans.** 7.89 kms^{-1}]

34. A satellite revolves around a planet in an orbit just above the planet's surface. Find the period of the satellite. [**Ans.** 4.2×10^3 s]

35. Determine the velocity with which a body must be thrown vertically upward from the surface of earth so that it may reach a height of $10R$, where R is the radius of the earth.
$\left[\textbf{Ans.} \left(\frac{20\,GM}{11\,R}\right)^{1/2}\right]$

36. Prove that the gravitational field and gravitational potential at any point on the surface of the earth are g and gR, respectively. The earth may be assumed to be a sphere of uniform density.

37. A meteor is falling. How much gravitational acceleration would it experience when its height from the surface of the earth is equal to three times the radius of the earth? $\left[\textbf{Ans.} \frac{g}{16}\right]$

LONG ANSWER Type II Questions

38. The planet Neptune travels around the Sun with a period of 165 yr. Show that the radius of its orbit is approximately thirty times that of the earth's orbit, both being considered as circular.
[**Ans.** $R_2 = 30R_1$]

39. A 70 kg boy stands 1 m away from a 60 kg boy. Calculate the force of gravitational attraction between them.
[**Ans.** 2.7972×10^{-7} N]

40. At what height above the surface of the earth, value of acceleration due to gravity is 36% of its value on the surface of the earth? Given radius of the earth = 6400 km.
[**Ans.** 4267 km]

41. A particle is fired vertically upward with a speed of 15 km/s. Find the speed of particle when it goes out of the earth's gravitational pull. [**Ans.** 10 km/s]

42. Calculate the period of revolution of the neptune around the sun. Given that radius of its orbit is 30 times the earth's orbital radius around the sun. [**Ans.** 164.3 yr]

43. Two steel balls whose masses are 5.2 kg and 0.25 kg are placed with their centres half a metre apart with what force do they attract each other? [**Ans.** 3.468×10^{-10} N]